Autonomy-Based Security Design: The Allocation Of Cash Flow And Control Rights

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Abstract

We derive debt, equity, convertible debt and asset-backed debt securities as optimal security designs in an environment in which the owner-manager has an endogenous control preference – a desire for autonomy – arising from the anticipation of future disagreement with investors over a value-maximizing project choice. This disagreement is engendered not by agency or asymmetric information problems but by heterogeneous rational beliefs about the precision of a public signal of project value. Optimal security design seeks to simultaneously achieve an efficient allocation of cash-flow sharing rights and control rights over project choice. In some circumstances, the optimal security design involves a linear cash-flow sharing rule and joint control between the manager and investors, resembling common equity. In fact, whenever joint control is efficient, the optimal security is always equity. In other circumstances, the optimal security design involves promising investors a fixed amount, with exclusive managerial control. This resembles riskless debt in some circumstances and risky debt in others. We also identify the circumstances in which convertible debt is the optimal security, and when it will be efficient for the firm to segregate (securitize) a group of assets and then issue a debt claim against those assets as in the case of asset-backed commercial paper. Our analysis also explains why firms tend to issue equity when their stock prices are high.

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1 INTRODUCTION

The voluminous literature on security design has greatly advanced our understanding of the circumstances in which various financial securities arise endogenously in response to the risk-sharing needs of agents or informational frictions of various sorts. Each financial security is characterized by two attributes: cash-flow sharing rights and control rights. Cash-flow sharing rights refer to how the total cash flow should be divided among different types of securities, whereas control rights determine who has control over decisions that affect the probability distribution of cash flows. The existing research on cash-flow sharing rights has been primarily concerned with how different allocations of cash-flow rights can minimize the impact of informational and agency problems and/or provide improved risk sharing in incomplete markets (e.g., Allen and Gale [1988], Boot and Thakor [1993], Chowdhry, Grinblatt and Levine [2002], and Duffie and Rahi [1995]). In the literature on the optimal allocation of control rights, the goal is to determine how to allocate control so as to minimize losses in investment efficiency or facilitate efficient continuation/investment decisions (e.g., Aghion and Bolton [1992], Gale and Hellwig [1985], and Townsend [1979]).

Despite these advances, there is much we do not know. For example, are the cash-flow sharing rules observed with debt and equity linked in an economically natural way with the control allocations embedded in these contracts?[1] For example, equity is typified by *joint control* over real decisions between the decision-maker ("manager" henceforth) and shareholders. We define "joint control" as shared control, whereby multiple parties share control and there is a predetermined rule to resolve disagreement over real decisions. Such control arrangements are ubiquitous with equity. Start-up firms give authority over corporate decisions to the board, and in the majority of venture-capital-financed firms, neither the venture capitalist nor the entrepreneur enjoys exclusive control (Kaplan and Stromberg [2003]). In large, publicly-traded firms, the manager exercises control jointly with the board of directors, some of whom are on the board due to their equity ownership stakes. That is, joint control is exercised by multiple controlling shareholders, including the manager. By contrast, control in debt contracts is allocated in an exclusive manner. Control rests entirely with the manager (representing the

[1] There is a literature on voting rights that asserts that the control associated with voting rights should be linked to a risky security (equity). See, for example, Grossman and Hart [1988].

shareholders) unless there is a violation of covenants or default on the debt. And debt contracts come in a rich variety of forms. The most common is straight debt, where creditors have a fixed prior claim on all of the firm's cash flow. However, sometimes firms segregate some of their assets and transfer them to a trust, so that debt claims secured only by these assets can be issued. An example of such securitization is asset-backed commercial paper (ABCP henceforth), which has been growing in popularity.[2] It is claimed that segregating a pool of assets to serve as collateral permits access to the commercial paper market to firms that would not otherwise have had this access.[3] Moreover, firms often use convertible debt, whereby investors hold debt and give the manager complete control until they decide to convert to equity and then they share control with the manager.

Why is equity, which represents a *linear* cash-flow sharing rule, characterized with *joint control*? Why is it that debt, which provides a payoff to the debtholder that is *concave* in the total cash flow, has associated with it an *exclusive allocation of control*? When do firms decide to use straight debt and when is ABCP preferred? Why do firms use convertible debt?

Beyond these fundamental issues related to the economic underpinnings of linkages between cash-flow sharing rules and control-right allocations, we have little understanding of the *timing* of security issuances by firms. There is an emerging empirical literature that has shown fairly convincingly that security issuances by firms are neither random (as in a Modigliani and Miller [1958] world), nor do they appear to correspond to the tradeoffs suggested by agency or information signaling models (see, for example, Baker and Wurgler [2002], Graham and Harvey [2001], and Welch [2004]). It appears from this evidence that firms tend to issue equity when their stock prices are high. While it has been conjectured that this may be because investors are irrational and managers are attempting to "time the market" (e.g., Baker and Wurgler [2002]), there are numerous papers that have provided evidence against the timing hypothesis (e.g., Butler, Grullon and Weston [2004], and Flannery and Rangan [2005]), so this issue remains unsettled. It is therefore useful to ask: is there something about the *design* of equity as an optimal security that generates this kind of issuance behavior?

Our purpose in this paper is to address these questions from a basic security design perspective. At first blush, it would be tempting to fall back on the familiar agency or asymmetric

[2]ABCP typically involves bank-linked conduits that involve short-term assets. Single-seller conduits exist, but are less common than multiple-seller conduits in which multiple firms pool their assets together as collateral to back up their commercial paper issues.

[3]Since such securitization often involves pooling assets across many firms, it also serves to reduce asymmetric information, as shown by Gorton and Pennacchi [1990]. This feature does not play a role in our analysis, however, because we do not consider such pooling.

information arguments to come up with explanations. However, these approaches are unlikely to bear fruit here. Asymmetric information problems can be resolved through direct truth-elicitation mechanisms, employing the Revelation Principle (see Myerson [1979]), so that control issues would be rendered moot.[4] If problems of agency (e.g., Holmstrom [1979], and Jensen and Meckling [1976]) or private managerial control benefits (e.g., Aghion and Bolton [1992]) are predominant, then control would *not* be joint. Rather, it would rest entirely with the investors in an ex ante efficient allocation of control in order to minimize agency costs ex ante.[5] Moreover, while convertible debt can attenuate the asset-substitution moral hazard that is associated with straight debt (e.g., Green [1984]), a simpler way to eliminate this moral hazard is simply to use equity. That is, an approach that explains why convertibles are used, conditional upon the firm deciding to use some form of debt, cannot explain, based on the principles of security design, why the firm chooses to use any form of debt in the first place.

Our analysis takes a different route. In our analysis, an *endogenous* preference for control arises because of the possibility of fundamental disagreement between the manager and investors. In practice, control by investors manifests itself in various ways: restrictions on the size and composition of board of directors, management retention/firing and compensation decisions, the board's veto rights on management's investment decisions and proxy fights, etc. The agency perspective is that such control should be designed to curb self-serving managerial behavior. However, in some circumstances, the manager may actually be attempting to maximize firm value, but may find investors disagreeing with her over what maximizes firm value. This kind of *disagreement* may cause investors to block the manager from making decisions that the manager believes are value-maximizing. There must therefore be an ex ante allocation of control that determines how such disagreement should be resolved. We define managerial "autonomy" as the control allocated to the manager in terms of the probability with which she can decide when there is disagreement with investors.[6] A situation in which this probability is in the interior of $(0, 1)$ is one in which managerial autonomy is partial and there is joint control between the manager and financiers. The manager has an endogenous preference for

[4] Of course, such resolutions would be second best, involving efficiency losses.

[5] Burkart, Gromb and Panunzi [1997] show that it may be optimal to leave some discretion with management. However, in their model the *ex post* allocation of control is never joint. It always rests either with the manager or the investor, depending on who is better informed (see also the modeling of "real authority" in Aghion and Tirole [1997]).

[6] We have viewed equity as a one-share, one-vote contract, so joint control is interpreted as being implemented through the corporate governance structure. This is the standard view in the literature. For example, in Fluck [1999], management dismissal is the principal mechanism by which shareholders, all of whom have voting power, exercise control. The manager's ownership stake defines the allocation of control across the manager and outside shareholders.

autonomy because it facilitates decisions that she believes will increase firm value. However, such managerial autonomy is costly to investors, because from their perspective, giving the manager more autonomy is equivalent to providing her greater leeway to make bad decisions. Consequently, greater managerial autonomy increases the cost of capital to the firm. It is this tradeoff between the endogenous desirability of autonomy for the manager due to the decision-making elbow room it provides on the one hand and the higher cost of capital associated with greater autonomy on the other hand that serves as the starting point of our analysis.

We develop this idea in the context of a model in which the owner-manager of a firm that has assets in place that will produce a state-contingent future cash flow whose probability distribution is agreed upon by everyone. The firm also raises external financing for a project that it will implement in the future. After raising the financing but prior to the decision of whether to invest in the project, a signal is generated about the value of the project. While the signal is publicly observed without ambiguity by all agents, the manager and investors randomly draw potentially different (but possibly correlated) prior beliefs about the precision of the signal. This causes them to have potentially different posterior assessments of the value of the project, and disagreement may arise over whether the project should be accepted or rejected.

This disagreement plays a key role in the model and we introduce it through heterogeneous prior beliefs which are assumed to be rational in the sense of Kurz [1994a,b]. That is, these beliefs are all consistent with the historical data. Although the Harsanyi doctrine calls for uniform prior beliefs,[7] economic theory stipulates rationality as dealing only with the revision of prior beliefs and does not address where these priors come from. They are to be viewed as part of the primitives, along with preferences and endowments. Kreps [1990] argues that heterogeneous priors represent a more general specification than uniform priors, and Morris [1995] explains that heterogeneous priors are consistent with Bayesian rationality. Numerous papers have employed heterogeneous prior beliefs, including Abel and Mailath [1994], Allen and Gale [1999], Boot, Gopalan and Thakor [2005], Garmaise [2001], Manove and Padilla [1999], Van den Steen [2004] and the famous Arrow-Debreu-Mckenzie model, and empirical evidence of heterogeneous beliefs and diverse interpretations of the same information can be found in Kandel and Pearson [1995] and Tagaki [1991].

This approach allows us to generate numerous results about the optimal design of the securities with which the owner-manager finances the project. First, we show that when the

[7] Samuelson's [2004] summary of the justification for the uniform prior beliefs assumption is that it is just "a welcome source of modeling discipline."

firm has no assets in place, it is optimal for the manager to finance with a contract that involves a linear cash-flow sharing rule and joint control with financiers that gives the manager *partial* autonomy. That is, equity represents the optimal security design. The essence of the intuition here is that a linear cash-flow sharing rule, by aligning the objectives of the manager and investors, *minimizes* potential difference of opinions over the value-maximizing project choice and hence optimizes the allocation of control rights. Second, we show that the value of the firm's equity is an increasing function of the degree of agreement between the manager and investors, implying that the attractiveness of issuing equity and financing the project is higher when the firm's stock price is higher. Third, when the firm's assets in place have a relatively high known value, it is optimal for the firm to raise financing with a security that gives the manager *complete* autonomy. Moreover, this security has a cash-flow sharing rule that is concave, providing investors a fixed payment as long as the firm's cash flow exceeds that payment, and the firm's entire cash flow if it falls below that promised fixed repayment. That is, a debt contract is optimal. This kind of cash-flow sharing rule minimizes the cost to the manager of retaining complete autonomy over project choice when the assets in place produce a high cash flow. Fourth, when the firm's assets in place have a lower but known value, the firm uses a mix of debt and equity as optimal securities. Finally, when the firm's assets in place have sufficient uncertainty about their value, it is optimal for the firm to use either convertible debt or ABCP. Convertible debt is preferred either when the fixed cost of setting up an ABCP program is sufficiently high or when this cost is low but the firm's stock price is relatively high. If the realized value of the firm's assets in place turns out to be high, investors hold on to their debt and do not convert to equity; this can be interpreted as a situation in which the value of the firm's debt securities is high relative to its stock price. If the realized value of the firm's assets in place turns out to be low, investors convert their claim to equity. ABCP is deployed when the fixed cost of deploying it and the firm's stock price are sufficiently low.

Our analysis reveals an interesting aspect of the choice between equity and debt. It shows that this choice relies in a significant way on the choice between joint and exclusive control. Whenever joint control is efficient, the optimal security design is equity. Whenever debt is the optimal security design, exclusive (managerial) control is efficient. Whenever there is sufficient uncertainty ex ante about whether joint or exclusive control will be preferred ex post, either convertible debt or ABCP is the optimal security design. The cash-flow sharing rules accompanying debt, equity, convertible debt and ABCP are simply integral parts of the control allocations.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the base model. The analysis of the base model is contained in Section 4. Section 5 discusses model extensions and empirical predictions. Section 6 concludes. All proofs are relegated to the Appendix.

2 RELATED LITERATURE

Our paper is related to the rapidly-grown literature on security design (see Allen and Winton [1995] and Duffie and Rahi [1995] for surveys). While this literature can be broadly categorized into papers dealing with the allocation of cash-flow rights and papers dealing with the allocation of control rights, there are many strands within each category. In the category of papers concerned primarily with the optimal allocation of cash-flow rights, some have dealt with informational frictions as the driving force. Boot and Thakor [1993] demonstrate that splitting the claim to the total cash flow of an asset into two separate securities, one informationally sensitive and the other less so, promotes information acquisition and informed trading, thereby maximizing the issuer's revenue. They also provide an explanation for securitization with multiple tranches within this framework. Other contributions to this strand include, among others, DeMarzo [2005], Fulghieri and Lukin [2001], and Gorton and Pennachi [1990]. DeMarzo and Duffie [1999] examine how an issuer should design and sell a security backed by specific assets, i.e., the optimal design of securitization. The issuer, who has private information about the assets that back the security, faces a tradeoff in deciding which assets to include in the portfolio backing up the security. Including an asset in the portfolio makes the security less liquid due to the issuer's private information, whereas excluding it imposes a retention cost of holding the asset rather than selling a claim against it. They show the circumstances in which the optimal security is a standard debt contract. Chiesa [1992] shows that the use of warrants with debt can lessen the effort-provision inefficiencies encountered with straight debt. In a rational expectations framework with exponential utility and normal distributions, Rahi [1996] shows that issuers prefer equity because it leads to a fully-revealing rational expectations equilibrium and hence does not exploit the issuer's informational advantage. By contrast, Garmaise [2001] shows that departing from rational expectations leads to the optimality of prioritized tranches and risky securities that are issued even by firms that are not cash-constrained. Chowdhry, Grinblatt and Levine [2002] demonstrate the optimality of currency swaps on the basis that they allow the priorities of claims in bankruptcy to switch depending on the exchange rate. Another strand of the literature on allocating cash-flow rights has dealt with spanning and risk sharing considerations in incomplete markets. Contributions to this strand include Allen and Gale [1988], Madan and Soubra [1991], and Pesendorfer [1995].

The second category of security design papers deal with allocating control rights over corporate decisions. We will briefly discuss a few relevant papers. One strand of this literature, called "costly state verification" models, has established the optimality of a debt contract on the grounds that when the cash flow that the security represents a claim against is not directly observable or contractible, it is efficient to promise investors a fixed payment independent of the cash flow. The firm is induced to meet this promised repayment either by use of a threat to transfer ownership of assets to the investors who can then take possession of the entire cash flow or through a threat to withhold additional funds (see, for example, Bolton and Scharfstein [1990,1996], Gale and Hellwig [1985], Hart and Moore [1998], and Townsend [1979]).[8] Thus, control rights transfer to the investors upon default, as they do with a debt contract. Boyd and Smith [1999] extend this framework to generate contracts that they interpret as debt and equity. Aghion and Bolton [1992] consider a wealth-constrained entrepreneur, with private control benefits, seeking funding from outside investors to finance a project. They show that in an optimal allocation, control rests invariably with one party, and such a control allocation can be achieved by using debt with an appropriate bankruptcy institution. Other contributions to the control aspects of security design include Berglof and von Thadden [1994], Berkovitch and Israel [1996], Dewatripont and Tirole [1994], Kalay and Zender [1997], and Zender [1991].

There are four key differences between the existing literature and our work. First, the existing literature typically deals with the allocation of cash-flow rights and the allocation of rights over control of investment decisions as two distinct problems. We view them essentially as two sides of the same coin and endogenize both as part of the same security design problem. Second, joint control does not emerge as part of an optimal security design in the existing literature, whereas it is a central element of the design of equity in our analysis. Third, unlike the existing literature, we simultaneously characterize equity, straight debt, convertible debt and ABCP as optimal securities even when investors in any security have sufficient wealth to provide all the financing the firm needs, and we characterize the circumstances in which the manager will use each type of security. Finally, unlike the existing literature, we use a security design framework to show that it is optimal for the manager to issue equity when the firm's stock price is high and equity has been endogenously derived as an efficient security.

[8]The optimality of debt in the presence of asset-substitution moral hazard has been shown by Povel and Raith [2004].

3 THE MODEL

In this section, we describe the model, including the agents, the economic environment, the potential disagreement between the manager and the investor, and managerial autonomy. Because we view investors as a homogenous group, we will refer to them henceforth as a single investor.

3.1 The Agents and Economic Environment

We assume universal risk neutrality and a zero riskfree interest rate. We consider a four-date economy ($t = 0, 0.5, 1$ and 2) that consists of one firm and one investor. The firm is run by an owner-manager (manager hereafter) who has no capital except her complete ownership of the firm. The firm also has assets in place (AIP) at $t = 0$ that will produce a cash flow, $\tilde{F}$, at $t = 2$. Viewed at $t = 0$, $\tilde{F}$ is a random variable that has a density function $g(\cdot)$ with support $[0, \infty)$. This density function is common knowledge at $t = 0$. At $t = 0.5$, the value of $\tilde{F}$ that will be realized at $t = 2$ becomes common knowledge. At $t = 0$, the firm also has a project that requires a \$1 investment. The firm has no internal funds and thus has to raise \$1 from the investor. The manager designs the security at $t = 0$ to raise the required capital. The project can be one of two types: good (G) and bad (B). The common prior belief at $t = 0$ is that with probability $\gamma \in [0, 1]$ the project is good, and with probability $[1 - \gamma]$ the project is bad. A good project pays off $H > 1$ for sure at $t = 2$, whereas a bad project always pays off zero. We assume that $\gamma H < 1$, i.e., *a priori* the project has a negative expected NPV.

3.2 Potential Disagreement between the Manager and the Investor

At $t = 1$, the firm must decide whether to invest in the project. This decision is based on a public signal that is observed by both the manager and the investor. The signal is $s \in \{s_G, s_B\}$, where s_G is a good signal and s_B is a bad signal. Everybody sees the same signal, i.e., there is no disagreement regarding the signal itself. Moreover, we assume that the prior probabilities are $\Pr(s = s_G) = \gamma$ and $\Pr(s = s_B) = [1 - \gamma]$.

Although the manager and the investor see the same signal and have the same prior beliefs regarding the value that the signal will take (s_G or s_B), they have different prior beliefs regarding the precision of the signal. The precision of the signal, which we denote as q, can be of three types: precise (P), not-precise (N) and uninformative (U). The probabilities of drawing P, N and U are θ_P, θ_N and θ_U, respectively, with $\theta_P + \theta_N + \theta_U = 1$ and $\theta_j \in (0, 1), \forall j \in$

$\{P, N, U\}$. A precise signal is viewed as perfect and causes the receiver of the signal to come up with a posterior belief that puts all of the probability weight on the value of the signal, a not-precise signal is viewed as noisy but informative and causes the posterior belief about project NPV to be a weighted average of the prior belief and the signal, and an uninformative signal is disregarded so that the posterior belief about project quality stays at the prior belief.

To see this concretely, consider the case in which the signal is s_G. When the prior belief about signal precision is P, the agent's belief about project quality is $\Pr(G|s = s_G, q = P) = 1$; when the prior belief about signal precision is N, the agent's belief about project quality is $\Pr(G|s = s_G, q = N) = \hat{\gamma} \in (\gamma, 1)$; and when the prior belief about signal precision is U, the agent's belief about project quality remains at its prior, i.e., $\Pr(G|s = s_G, q = U) = \gamma$. Moreover, we assume that $\hat{\gamma}H = 1$, i.e., the NPV of the project is perceived to be zero when the signal is s_G and it is not-precise. If the signal is s_B, it is clear that a precise signal leads the agent to believe that the project is bad almost surely, a not-precise signal causes the posterior belief to fall below the prior and leads the agent to believe that the NPV of the project is negative, and an uninformative signal doesn't change the agent's prior belief about the NPV of the project, which is again negative. Thus, a signal realization $s = s_B$ results in agreement between both agents at $t = 1$ that the project has negative NPV, regardless of prior beliefs about signal precision. With this structure, a signal s_B always leads to rejection of the project, whereas a signal $s = s_G$ may lead to different decisions depending on the precision of the signal.

We model the potential divergence of prior beliefs between the manager and the investor regarding the signal precision as follows. They randomly draw prior beliefs $q_m, q_i \in \{P, N, U\}$ about the precision of the signal, where q_m and q_i are the prior beliefs about signal precision of the manager and the investor, respectively. Throughout we will use the subscript "m" to denote the manager and the subscript "i" to denote the investor. Thus, the prior beliefs of signal precision may be different and we assume the correlation structure to be:

$$\Pr(q_i = P|q_m = P) = \rho \in [0, 1], \tag{1}$$

$$\Pr(q_i = N|q_m = P) = [1-\rho]\left[\frac{\theta_N}{\theta_N + \theta_U}\right] \equiv [1-\rho]\tau, \tag{2}$$

$$\Pr(q_i = U|q_m = P) = [1-\rho]\left[\frac{\theta_U}{\theta_N + \theta_U}\right] \equiv [1-\rho][1-\tau], \tag{3}$$

$$\Pr(q_i = P|q_m = N) = [1-\rho]\left[\frac{\theta_P}{\theta_P + \theta_U}\right] \equiv [1-\rho]\hat{\tau}, \tag{4}$$

where $\tau \equiv \theta_N/[\theta_N + \theta_U]$ is the relative likelihood that the signal is not-precise conditional on it being either not-precise or uninformative, and $\hat{\tau} \equiv \theta_P/[\theta_P + \theta_U]$ is the relative likelihood that the signal is precise conditional on it being either precise or uninformative.

The value of ρ measures the "degree of agreement" between the manager and the investor. The higher is ρ, the greater is the agreement between the manager and the investor in the sense that the higher is the probability that their prior beliefs about signal precision will coincide. A value of $\rho = 1$ indicates perfect agreement and a value of $\rho = 0$ indicates perfect disagreement. The agreement parameter ρ is affected by the attributes of the firm's project. If the project involves a radically new product or business design, there may be little hard historical data to gauge the probability of the project succeeding in the future. Project evaluation may thus have to be based largely on soft information that is inherently subjective in nature (Stein [2002]), possibly causing ρ to be low. By contrast, for a project that is somewhat more familiar in the sense that similar projects have been tried in the past, there may be a more balanced mix of hard historical data and soft information, so the value of ρ may be relatively high.

We assume that even though agents have heterogeneous beliefs about the precision of the signal, all agents have "rational beliefs" as defined by Kurz [1994a,b]. Kurz shows that rational beliefs, which may be heterogeneous to start with, need not converge to a unique distribution even with countably infinite common observations of past data. That is, rational beliefs are different from rational expectations. Although Kurz's foundation of rational beliefs has many aspects, the two aspects most relevant for our analysis are that agents have different priors and that all these priors are consistent with the data in the sense that none can be precluded by historical data.

Technically, what we are modeling is a setting in which the economic observables based on which agents form beliefs about the signal precision are "stable" but not "stationary" (see Kurz [1994a,b]).[9] In this setting, the rational expectations hypothesis requires agents to have information about underlying processes that cannot be derived from historical data, whereas the rational beliefs hypothesis requires only that their beliefs be consistent with the data. We believe that for new projects that are dissimilar in nature compared to the kinds of projects the firm has been involved with in the past and for which there is a paucity of hard historical data, it is reasonable to posit that prior beliefs about the precision of an interim signal about the project are derived from non-stationary economic variables. Consequently, agents will *not* be able to *uniquely* derive the precision of the signal from historical data, and thus many different distributions of precision may be consistent with the data. In such a setting, our assumption

[9] Let $(\mathbf{R}^\infty, \mathcal{F}, P, T)$ represent a stochastic dynamic system, where $\mathcal{F}$ is the σ-algebra of $\mathbf{R}^\infty$, P is a probability measure, and T is the shift transformation on $\mathbf{R}^\infty$. The system is said to be stable if for every cylinder $\mathcal{C} \in \mathcal{F}$, $\lim_{n\to\infty} \frac{\sum_{k=0}^{n-1} 1_{\mathcal{C}}(T^k(x))}{n}$ exists for P-almost every $x \in \mathbf{R}^\infty$. The system is stationary if the shift transformation T is measure preserving, i.e., for all $\mathcal{C} \in \mathcal{F}$, $P(T^{-1}\mathcal{C}) = P(\mathcal{C})$, where $T^{-1}\mathcal{C} \equiv \{x : Tx \in \mathcal{C}\}$.

that agents have rational beliefs, that do not necessarily conform to rational expectations, seems reasonable.

3.3 Managerial Autonomy and the Allocation of Control Rights

Whenever the manager and the investor disagree over the project investment decision, there has to be a rule to resolve the disagreement. We model this using the concept of "autonomy" for the manager. Managerial autonomy is the degree of control given to the manager and it is represented by the probability $\eta \in [0, 1]$ that the manager will be able to make the project investment decision she thinks is best for the firm in the face of disagreement with the investor. A higher η means higher managerial autonomy. Thus, $[1 - \eta]$ is the probability with which the investor can overrule the manager's investment decision. *Ceteris paribus* the manager prefers more autonomy to less, even though she derives no direct utility from autonomy, i.e. there is no exogenous private control benefit in this model. The key is that η is an important choice variable in the manager's security design at $t = 0$, and affects the manager's project choice at $t = 1$. The only observable at $t = 1$ is the value of the AIP, so η may be conditioned on this value. That is, $\eta : [0, \infty) \to [0, 1]$.

A premise here is that if the manager does not want to invest, the investor will not force or bribe her to propose the investment. To justify this premise, suppose managerial effort is essential for the project to produce any cash flow. The manager can choose a costly effort $e \in \{0, 1\}$, resulting in a project cash flow of ez, where $z = H$ for project G and $z = 0$ for project B. Thus, if the manager chooses $e = 0$, the project payoff will be 0 for sure regardless of its type. If the manager chooses $e = 1$, the project payoff depends on its type. For project G, a choice of $e = 1$ leads to a cash flow of H, whereas for project B a choice of $e = 1$ leads to a cash flow of 0 (the same cash flow obtained with $e = 0$). The manager's effort choice is unobservable and unverifiable. Let the disutility of effort for the manager be $e\zeta$, where $\zeta > 0$. Thus, if $w(e)$ is the manager's payoff associated with an effort choice of e, incentive compatibility requires that $e \in \text{argmax}[w(e) - e\zeta]$, and if the manager is to be induced to choose $e = 1$, then we need

$$w(1) - \zeta \geq w(0). \tag{5}$$

We assume that $\gamma H < \zeta < 1$. Given this, it will never be optimal for the investor to bribe the manager to take the project when the manager strictly prefers to reject it. To see this, note that the manager has a strict preference to reject the project when $s = s_B$ or when $s = s_G$ and she draws a signal precision of U. In the former case, both the manager and the investor agree that the project is of negative NPV and should be rejected. In the latter case, the manager assesses the value of the project as γH if she chooses $e = 1$. Since the manager can never receive more than the entire project cash flow, $\gamma H < \zeta$ ensures that the manager will never choose $e = 1$ even if the investor bribes her to invest.[10] This means the manager will never want the project in this case since an effort choice of $e = 0$ guarantees a cash flow of 0 regardless of the investor's belief about the project type. But since $\zeta < \hat{\gamma} H = 1$, a signal of precision N with $s = s_G$ may permit the investor to induce the manager to choose $e = 1$ without offering a bribe. Hence, the only case in which project investment is unambiguously precluded is when the manager strictly prefers not to invest in the project.

To minimize notation and keep the analysis as simple as possible, we will suppress managerial effort for much of the remainder of the analysis and will simply assume that only those projects that the manager wishes to propose to the investor can be accepted or rejected. That is, the manager has the first move and can unilaterally reject a project she does not like, but requires investor approval for acceptance for those projects she proposes.

The sequence of events is summarized in *Figure 1.*

[Figure 1 goes here]

3.4 The Allocation of Cash-Flow Sharing Rights and Security Design

Let α represent the fraction of the firm's cash flow (denoted as V) at $t = 2$ that is given to the investor. This fraction can depend only on the observables at $t = 2$: the realized value of the AIP (F) and the project cash flow (denoted as Z). That is, $\alpha : [0, \infty) \times \{0, 1, H\} \rightarrow [0, 1]$, where $F \in [0, \infty)$ and $Z \in \{0, 1, H\}$, where we refer to 1 as the value of Z when there is no project investment. Note that $V = F + Z$. We assume that it is costless to allocate cash-flow rights over total firm cash flows. However, if a portion of the firm's assets is separated from the rest of the firm and cash-flow rights are allocated exclusively against that portion, there is a fixed transaction cost, S, involved in doing so. The practical interpretation of S is that it represents the legal and administrative costs of setting up a securitization program, such as ABCP.

Designing a security means choosing a pair of functions $\{\alpha, \eta\}$. In what follows, we will show that these functions are simultaneously chosen as part of efficient security design.

[10]To see this, note that since $w(1) \leq \gamma H < \zeta$, we have $w(1) - \zeta < w(0) = 0$. Thus, the manager will not propose the investment voluntarily. If the investor bribes the manager by giving her an additional compensation of b, the manager still strictly prefers to choose $e = 0$, since $w(1) - \zeta + b < w(0) + b$.

4 THE ANALYSIS

In this section, we study the manager's optimal security design problem at $t = 0$. We first characterize the optimal contract when the AIP take the value of zero for sure and then derive the optimal securities corresponding to different values of the AIP when the value of the AIP is positive and deterministically known at $t = 0$. We then allow the value of the AIP to be stochastic to examine its implications for optimal security design.

4.1 Optimal Security Design when the Value of the AIP Is $\mathbf{F = 0}$

In this section, we first show that equity, which is characterized by a linear cash-flow sharing rule, is the optimal security design when the firm has no AIP. We then examine the optimal control rights allocation with equity.

A. The Optimality of Equity as a Cash-Flow Sharing Rule

Suppose the sharing rule is that the investor gets $\Phi(Z) \equiv [\alpha_Z \times Z]$ at $t = 2$, where Z is total value of the project cash flow (including the initial \$1 investment), and α_Z is the value α takes when the project cash flow is Z. Note that $V = Z$ since $F = 0$. We derive the conditions under which it is optimal for the manager to choose $\Phi(\cdot)$ to be linear in Z. Note that there are only three possible values for the project cash flow at $t = 2$: $Z \in \{0, 1, H\}$, where $Z = 0$ corresponds to an investment in a bad project, $Z = 1$ corresponds to no investment, and $Z = H$ corresponds to an investment in a good project. Thus, the sharing rule can be specified as giving the investor α_j share of the project cash flow when $Z = j$, for $j \in \{0, 1, H\}$. A linear sharing rule is characterized by $\alpha_H = \alpha_1 = \alpha_0$, a convex sharing rule is characterized by $\alpha_H > \alpha_1 > \alpha_0$, and a concave sharing rule is characterized by $\alpha_H < \alpha_1 < \alpha_0$. It turns out that in our three-state framework, the feasible set of cash-flow sharing rules is entirely spanned by the linear, convex and concave sharing rules.[11] This may seem rather special, but we show in Section 5 that our results generalize to a setting in which there is an arbitrarily large number of cash-flow states.

A1. The Manager's Basic Tradeoff: We first study the basic tradeoff faced by the manager. We use the linear sharing rule as an example, but the intuition applies to convex and concave

[11] As shown in the proof of Proposition 2, the value of α_0 does not matter for the optimality of a linear cash-flow sharing rule.

sharing rules as well. In a linear contract, let us assume $\alpha_j = \alpha$ for $\forall j \in \{0, 1, H\}$. Note that from the investor's perspective, the payoff to the investor (denoted as $\mathbf{E}_i(\Phi(Z))$) is:

$$\begin{aligned}
\mathbf{E}_i(\Phi(Z)) \quad = \quad & \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = P | q_m = P)[\alpha \times H] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = N | q_m = P)\big[\hat{\gamma}[\alpha \times H] + [1 - \hat{\gamma}][\alpha \times 0]\big] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = U | q_m = P)[\eta]\big[\gamma[\alpha \times H] + [1 - \gamma][\alpha \times 0]\big] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = U | q_m = P)[1 - \eta][\alpha \times 1] \\
& + \Pr(s = s_G) \Pr(q_m = N) \Pr(q_i = P | q_m = N)[\alpha \times H] \\
& + \Pr(\text{All other events})[\alpha \times 1]. \qquad (6)
\end{aligned}$$

The first term corresponds to the case in which $s = s_G$ and both the manager's and the investor's prior beliefs are that the signal is precise ($q_m = q_i = P$), and hence they agree to assign the project a value of H. The second term corresponds to the case in which $s = s_G$ and the manager's prior belief is that the signal is precise, while the investor's prior belief is that the signal is not-precise ($q_m = P, q_i = N$). The manager strictly prefers to invest in the project, while the investor is indifferent between acceptance and rejection.[12] We assume that the investor permits the manager to invest in this case. When $s = s_G$ and the manager's prior belief is that the signal is precise while the investor's prior belief is that the signal is uninformative ($q_m = P, q_i = U$), disagreement over the investment decision arises. The manager strictly prefers project acceptance, whereas the investor strictly prefers to reject the project. In this disagreement case, the manager gets to decide with probability η, and her decision to invest leads the investor to believe that his expected payoff is $\gamma[\alpha \times H] + [1 - \gamma][\alpha \times 0] = \alpha[\gamma H] < \alpha$. With probability $[1 - \eta]$, the investor gets to decide so that the project is rejected and the firm value remains at \$1. The third and fourth terms correspond to these two cases, respectively. The fifth term corresponds to the case in which $s = s_G$ and the manager is indifferent between acceptance and rejection of the project while the investor strictly prefers to invest and assigns a value of H to the project ($q_m = N, q_i = P$), so investment occurs. In all the other cases, the project is rejected and the firm value remains at \$1. These cases are summarized by the last term. For ease of exposition, we refer to each term as a *state*.[13]

The expected payoffs to the investor are different from the manager's perspective and from the investor's perspective. The manager's expectation of the payoff to the investor represents the firm's cost of capital from the manager's perspective and this determines the manager's

[12]Note that the expected payoff to the investor from the project if it is accepted is $\hat{\gamma}[\alpha \times H] + [1 - \hat{\gamma}][\alpha \times 0] = \alpha[\hat{\gamma} H] = \alpha$, which is the same as the payoff if the investor successfully blocks the investment.

[13]This is different from the three *cash-flow state*: $Z \in \{0, 1, H\}$.

utility and hence her security design choice. Let $\mathbf{E}_m(\Phi(Z))$ denote this cost of capital, and it is given by:

$$\begin{aligned}
\mathbf{E}_m(\Phi(Z)) &= \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = P|q_m = P)[\alpha \times H] \\
&+ \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = N|q_m = P)\,[\alpha \times H] \\
&+ \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = U|q_m = P)[\eta]\,[\alpha \times H] \\
&+ \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = U|q_m = P)[1-\eta][\alpha \times 1] \\
&+ \Pr(s = s_G)\Pr(q_m = N)\Pr(q_i = P|q_m = N)\big[\hat{\gamma}[\alpha \times H] + [1-\hat{\gamma}][\alpha \times 0]\big] \\
&+ \Pr(\text{All other events})[\alpha \times 1]. \qquad (7)
\end{aligned}$$

Comparing $\mathbf{E}_m(\Phi(Z))$ with $\mathbf{E}_i(\Phi(Z))$, we have:

$$\begin{aligned}
\mathbf{E}_m(\Phi(Z)) - \mathbf{E}_i(\Phi(Z)) &= \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = N|q_m = P)\big[[1-\hat{\gamma}][\alpha H]\big] \\
&+ \Pr(s = s_G)\Pr(q_m = P)\Pr(q_i = U|q_m = P)[\eta]\big[[1-\gamma][\alpha H]\big] \\
&- \Pr(s = s_G)\Pr(q_m = N)\Pr(q_i = P|q_m = N)\big[[1-\hat{\gamma}][\alpha H]\big]. \qquad (8)
\end{aligned}$$

Note that (8) represents the difference between the manager's and the investor's valuations of the investor's ownership stake. The investor's valuation is what is relevant for the investor's participation constraint, whereas the manager's valuation matters for her own expected utility calculation. The first term in (8) represents the expected difference in valuations when the manager strictly prefers to invest and the investor is indifferent ($\{s = s_G, q_m = P, q_i = N\}$). In that state, although the investor permits the manager to invest, that permission is not without cost to the manager. The second term in (8) represents the difference in valuations when the manager strictly prefers to invest and the investor wants to reject the project ($\{s = s_G, q_m = P, q_i = U\}$). It is the "price" that the manager has to pay to have autonomy $\eta > 0$. The third term in (8) is negative and it represents the difference in valuations when the manager is indifferent but the investor strictly prefers to invest in the project ($\{s = s_G, q_m = N, q_i = P\}$). In this state, the investor's valuation of the firm is greater than the manager's, so the manager gets a "bonus" in terms of a lower cost of capital.

In order to make the following analysis interesting, we assume that in a linear cash-flow sharing rule, the manager's valuation of the investor's ownership stake is always greater than \$1. Thus, the manager perceives a non-negative cost associated with the possibility of disagreement with the investor.[14] This is guaranteed by the following parametric assumption, which says that θ_N should not be too high relative to θ_P.[15]

[14]This implies that the manager perfers more agreement with investors to less agreement.

[15]Note that in a linear cash-flow sharing rule, $\mathbf{E}_i(\Phi(Z)) = 1$ and $\mathbf{E}_m(\Phi(Z)) = 1 + \gamma[1-\rho][\alpha H]\ [\theta_P \tau - \theta_N \hat{\tau}][1-\hat{\gamma}] + \theta_P[1-\tau][\eta][1-\gamma]$. It is clear that Assumption 1 is the sufficency parametric condition to ensure $\mathbf{E}_m(\Phi(Z)) \geq 1$.

Assumption 1.

$$\theta_N \hat{\tau} \leq \theta_P \tau.$$

The total firm value from the manager's perspective (denoted as $\mathbf{E}_m(Z)$) is:

$$\begin{aligned}
\mathbf{E}_m(Z) \quad = \quad & \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = P | q_m = P)[H] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = N | q_m = P)\,[H] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = U | q_m = P)[\eta]\,[H] \\
& + \Pr(s = s_G) \Pr(q_m = P) \Pr(q_i = U | q_m = P)[1 - \eta][1] \\
& + \Pr(s = s_G) \Pr(q_m = N) \Pr(q_i = P | q_m = N)[1] \\
& + \Pr(\text{All other events})[1]. \qquad (9)
\end{aligned}$$

Observe that the total firm value, as perceived by the manager, depends on the managerial autonomy parameter η and not on the sharing rule α. The manager's optimal security design is given by the solution to the following optimization problem at $t = 0$:

$$\begin{aligned}
& \max_{\{\alpha, \eta\}} \Big[\mathbf{E}_m(Z) - \mathbf{E}_m(\Phi(Z)) \Big], \\
& \text{s.t. } \mathbf{E}_i(\Phi(Z)) = 1. \qquad (10)
\end{aligned}$$

To see the tradeoff faced by the manager, note that although the firm value perceived by the manager, $\mathbf{E}_m(Z)$, is increasing in η, the cost of capital, $\mathbf{E}_m(\Phi(Z))$, is also increasing in η. The allocation of cash-flow rights does not affect firm value directly, but it affects the control rights allocation (as represented by the managerial autonomy parameter η) and the cost of capital. Thus, the allocations of cash-flow rights and control rights are inseparable ingredients in an optimal security design.

A2. The Optimality of a Linear Cash-Flow Sharing Rule: A sufficiency condition for the efficiency of a linear cash-flow sharing rule is that the probabilities of the two states, $\{s = s_G, q_m = P, q_i = N\}$ and $\{s = s_G, q_m = N, q_i = P\}$, are high enough. For the state $\{s = s_G, q_m = P, q_i = N\}$, the economic intuition is that the optimality of a linear sharing rule depends on it being sufficiently likely that there will be a state in which the manager strictly prefers to invest in the project and the investor is indifferent, because this is the state in which the congruence of objective functions between the manager and the investor due to the linear cash-flow sharing rule permits the manager to invest with probability 1 even though there is disagreement. If the cash-flow sharing rule were concave, investors would strictly prefer not to invest in this state and the manner in which joint control is allocated through the autonomy parameter would determine whether the manager can invest; the manager would perceive an efficiency loss in this state. For the state $\{s = s_G, q_m = N, q_i = P\}$, the economic intuition is that in this state the investor's valuation of the firm is higher than the manager's valuation. A higher probability of this state occurring results in the investor requiring a lower compensation and hence decreasing the cost of capital. Note that the occurrence probabilities of both the states depend on the value of θ_N. Thus, the condition given below is sufficient for a linear cash-flow sharing rule to dominate non-linear sharing rules.

Assumption 2. [16]

$$[1-\hat{\gamma}]\theta_N\hat{\tau} \in \Big[\bar{\eta}\big[[1-\gamma][1-\tau]+[1-\hat{\gamma}]\tau\big]\theta_P - \kappa[1-\bar{\eta}][1-\hat{\gamma}]\tau\theta_P, \bar{\eta}\big[[1-\gamma][1-\tau]+[1-\hat{\gamma}]\tau\big]\theta_P\Big],$$

where

$$\kappa \equiv \frac{[\gamma H]A_2}{1-\gamma A_2},$$

$$\bar{\eta} \equiv \frac{\sqrt{[A_1A_2]\left[\frac{A_1A_2}{1-A_1}+\theta_N[1-\rho]\hat{\tau}[1-\hat{\gamma}]\right]} - A_2\sqrt{\frac{1}{1-A_1}-H}}{\theta_P[1-\rho]\sqrt{[1-A_1]-H[1-A_1]^2}},$$

with $A_1 \equiv [1-\gamma][1-\tau]+[1-\hat{\gamma}]\tau$ *and* $A_2 \equiv \theta_P\rho + \theta_N[1-\rho]\hat{\tau}$.

We now have:

Proposition 1. *When $F = 0$ for sure, the optimal cash-flow sharing rule is linear in total project cash flow, whether the control allocation is joint or exclusive.*

The intuition is as follows. When there is a potential divergence of opinions between the manager and the investor, disagreement over project choice can arise for two reasons. One is due to a difference of opinions and the other is due to a difference of objective functions. Regardless of the reason, this disagreement represents a deadweight loss from the manager's standpoint. The manager seeks to design the security for raising capital in order to minimize this deadweight loss. There is nothing the manager can do about the divergence of opinions. But a linear cash-flow sharing rule eliminates the potential for disagreement due to a divergence of objectives. This permits the manager the autonomy to invest in the state in which she strictly prefers to invest and the investor is indifferent ($\{s = s_G, q_m = P, q_i = N\}$) under a linear sharing rule, without having to "pay" for that autonomy through a higher cost of capital.

[16] The restriction in Assumption 2, namely that $[1-\hat{\gamma}]\theta_N\hat{\tau} \leq \bar{\eta}\ [1-\gamma][1-\tau]+[1-\hat{\gamma}]\tau\ \theta_P$, is to ensure that the optimal managerial autonomy for a concave cash-flow sharing rule should not be too high, which permits the benefit of a linear sharing rule for the state $\{s = s_G, q_m = P, q_i = N\}$ to be large enough to support the optimality of linearity. This is because a high optimal managerial autonomy for a concave cash-flow sharing rule diminishes the relative advantage of linearity over concavity for the state $\{s = s_G, q_m = P, q_i = N\}$: in the limit if the optimal η for a concave cash-flow sharing rule is 1, i.e., exclusive managerial control, then the relative advantage of linearity over concavity for that state vanishes.

A security that makes the investor's payoff concave in the total cash flow is not optimal. The reason is that it dominates a linear sharing rule in one (cash-flow) state, but is dominated in the other two states. Assumption 2 guarantees that the overall effect is that linearity dominates. To see this, note that a concave sharing rule is better than a linear sharing rule for the manager in the high cash-flow state ($Z = H$), because the investor's share of the cash flow in this cash-flow state is lower with a concave sharing rule than with a linear sharing rule, i.e., $\alpha_H < \alpha$. This is due to the fact that the cost of capital is highest in the high cash-flow state since the manager's valuation of the firm may be higher than the investor's valuation due to disagreement. A lower α_H leads to a lower cost of capital in that cash-flow state. However, concavity presents drawbacks to the manager in two other states. First, the manager loses the ability to have complete autonomy in the state $\{s = s_G, q_m = P, q_i = N\}$ as discussed above, because the investor will now wish to reject the project and the autonomy parameter η will determine the manager's ability to invest in the project. Second, a lower α_H produces a lower benefit to the manager in the state $\{s = s_G, q_m = N, q_i = P\}$. In this state, the investor assigns a higher value to the firm than the manager does, so it is efficient for the manager – from an ex ante cost of capital standpoint – to give the investor as high an ownership as possible in this state. Concavity gives the investor a lower ownership stake in this state than linearity does. Note that the occurrence probabilities of both $\{s = s_G, q_m = P, q_i = N\}$ and $\{s = s_G, q_m = N, q_i = P\}$ are increasing in θ_N, so Assumption 2, which ensures that θ_N is large enough, suffices for linearity to dominate concavity.

The intuition behind why a linear sharing rule dominates one that provides the investor a share of the cash flow that is a convex function of the total cash flow is more straightforward. A convex sharing rule allocates a higher fraction of the firm's cash flow in the high cash-flow state ($Z = H$) than the linear sharing rule does, i.e., $\alpha_H > \alpha$. But disagreement between the manager and the investor arises precisely when the manager perceives the high cash-flow state $Z = H$ will occur but the investor thinks otherwise. In this cash-flow state, the manager values the firm more highly than the investor, so it is inefficient for the manager to give up a relatively high fractional ownership.

B. The Optimal Allocation of Control Rights for a Linear Cash-Flow Sharing Rule

We now examine the control rights allocation that is optimal with a linear cash-flow sharing rule. Let the linear sharing rule be one in which $\alpha_j = \alpha$ for $\forall j \in \{0, 1, H\}$ i.e., the investor receives $\Phi(Z) \equiv \alpha Z$ at $t = 2$ for some $\alpha \in [0, 1]$. That is, the investor receives a fraction $\alpha \in [0, 1]$ of the firm value and the manager retains the remaining $[1 - \alpha]$ ownership. The fraction α can be interpreted as the cost of capital for equity financing. The manager's problem at $t = 0$ is

$$\max_{\{\alpha,\eta\}} \mathbf{E}_m \left([1-\alpha]Z\right), \tag{11}$$

$$\text{s.t. } \mathbf{E}_i(\alpha Z) = 1. \tag{12}$$

The manager chooses the autonomy parameter η and the ownership fraction α to sell to the investor in order to maximize her expected terminal wealth (given by (11)), subject to the investor's participation constraint (12) that the investor's expected payoff (using his beliefs) equals the \$1 provided at $t = 0$ by the investor to the firm.

The value of the firm at $t = 0$ as assessed by the investor is given by:

$$\mathbf{E}_i(Z) = B_1 + B_2\eta, \tag{13}$$

and the manager's assessment of the value of the firm at $t = 0$ is given by:

$$\mathbf{E}_m(Z) = B_3 + \big[B_2[H-1]/[\gamma H - 1]\big]\eta, \tag{14}$$

where

$$\begin{aligned} B_1 &\equiv 1 + \gamma\big[\rho\theta_P + [1-\rho]\theta_N\hat{\tau}\big][H-1] > 1, \\ B_2 &\equiv \gamma\theta_P[1-\rho][1-\tau][\gamma H - 1] \in (-1, 0), \\ B_3 &\equiv 1 + \gamma\theta_P\big[\rho + [1-\rho]\tau\big][H-1] > 1. \end{aligned}$$

Thus, the cost of capital is $\alpha = 1/\mathbf{E}_i(Z)$. The manager's assessment of the value of her holdings is

$$[1-\alpha]\mathbf{E}_m(Z) = [1 - 1/\mathbf{E}_i(Z)]\,\mathbf{E}_m(Z). \tag{15}$$

We now have the following result:

Lemma 1. *The cost of capital α is increasing in the managerial autonomy parameter, η. The value of the manager's holding, as perceived by the manager, is decreasing in the cost of capital α for a fixed η, and increasing in η for a fixed α.*

This result describes the manager's tradeoff between managerial autonomy and the cost of capital with equity financing. Autonomy allows the manager to sometimes invest in a project that the investor finds unacceptable, so the investor requires compensation for this. This is why the cost of capital is increasing in managerial autonomy. However, for a fixed cost of capital, the manager values the firm more highly when η is higher because greater autonomy gives the manager greater ability to select projects she believes will maximize firm value. In choosing the optimal degree of autonomy, the manager trades off the value of autonomy for a given cost of capital against the higher cost of capital due to greater autonomy. Formally, the manager's problem can be formulated as:

$$\max_{\{\eta\}} [1 - 1/\mathbf{E}_i(Z)]\, \mathbf{E}_m(Z). \tag{16}$$

The manager solves for the optimal η, call it η^*, and then this determines α^* via (12). The optimal control-rights allocation in equity financing is given by the following proposition.

Proposition 2. *If* $\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]} \in (B_1 + B_2, B_1)$*, then the optimal control-rights allocation involves joint control between the manager and the investor, where the unique optimal managerial autonomy parameter is given by:*

$$\eta^* = \frac{\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]} - B_1}{B_2} \in (0, 1). \tag{17}$$

The corresponding ownership share sold to the investor is given by:

$$\alpha^* = \frac{1}{\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]}}. \tag{18}$$

If $\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]} \leq [B_1 + B_2]$*, then the optimal managerial autonomy parameter is* $\eta^* = 1$*. The corresponding ownership share sold to the investor is given by:*

$$\alpha^* = \frac{1}{B_1 + B_2}. \tag{19}$$

If $\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]} \geq B_1$*, then the optimal managerial autonomy parameter is* $\eta^* = 0$*. The corresponding ownership share sold to the investor is given by:*

$$\alpha^* = \frac{1}{B_1}. \tag{20}$$

This proposition establishes the optimality of joint control and exclusive control when a linear cash-flow sharing rule is used. The intuition is as follows. Note that $[B_1 + B_2] = \mathbf{E}_i(Z)|_{\eta=1}$ and $B_1 = \mathbf{E}_i(Z)|_{\eta=0}$. That is, $[B_1 + B_2]$ and B_1 are the investor's assessments of the firm value when the managerial autonomy parameter is set at its two extremes, $\eta = 1$ and $\eta = 0$, respectively. It is easy to see that $[B_1 + B_2] < B_1$ since $B_2 < 0$. $\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]}$ can be thought of as measuring the relative attractiveness of the project. The result says that in order to get an interior solution $\eta^* \in (0, 1)$ and hence joint control, the attractiveness of the project should not be too high or too low. If the project is sufficiently unattractive, i.e., $\sqrt{B_1 + B_3[1 - \gamma H]/[H-1]} \leq [B_1 + B_2]$, then the relative attractiveness of the project to the investor is sufficiently low, so it is efficient for the manager to have complete autonomy, i.e., $\eta^* = 1$. To understand this, note that the investor's valuation of the firm is lowest when the manager has complete control ($\eta = 1$) and highest when the investor has complete control

($\eta = 0$). When the project is sufficiently unattractive, the investor does not care very much about the project payoff and hence the control rights allocated to him, so the manager finds it relatively inexpensive to retain complete control; the impact of this in increasing the cost of capital is relatively small and is thus outweighted by the benefit of greater managerial autonomy to the manager. If $\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]} \geq B_1$, the project is sufficiently relatively attractive to the investor. Consequently, the cost of capital increases very steeply as managerial autonomy increases, and the manager finds it very expensive to retain any control for herself, making it efficient to surrender all control to the investor, i.e., $\eta^* = 0$.

Recall that Assumption 1 states that θ_N should not be too high. It turns out that Assumption 1 is also sufficient to guarantee that disagreement is costly to the investor, and lead to the following result:

Lemma 2. *The cost of capital, α^*, is decreasing in the agreement parameter ρ. The manager's net valuation of the firm, $[1 - \alpha^*]\mathbf{E}_m(Z)|_{\{\eta = \eta^*\}}$, is increasing in ρ.*

This result is readily interpretable. A high agreement parameter leads the investor to assign a higher value to the firm *ceteris paribus*, and hence demand a lower cost of capital.[17]

4.2 Optimal Security Design with Deterministic Value of AIP F > 0

We now analyze the case in which the value of the AIP is positive and deterministically known at $t = 0$. We distinguish two possibilities: $F \geq 1$ and $F \in (0, 1)$.

A. The Value of the AIP Is $\mathbf{F \geq 1}$

We now analyze the case in which the value of the AIP is $F \geq 1$ for sure. It follows now that riskless debt is the optimal security design in this case. Since the value of the AIP is $F \geq 1$, a debt contract that gives the investor a priority claim over the AIP ensures that the investor's repayment of his \$1 investment is guaranteed. Thus, the investor is indifferent to

[17] A necessary condition for Lemma 2 to hold is (see the proof of Lemma 2 in the Appendix)

$$\theta_N \hat{\tau} < [[1 - \tau][\hat{\gamma} - \gamma]/[1 - \hat{\gamma}] + 1]\theta_P.$$

Note that $\theta_P \tau < [[1 - \tau][\hat{\gamma} - \gamma]/[1 - \hat{\gamma}] + 1]\theta_P$. Thus, that necessary condition is guaranteed by Assumption 1. To understand that necessary assumption, note that its left-hand-side (LHS) corresponds to the "bonus" to the manager in the state $\{s = s_G, q_m = N, q_i = P\}$ in which the investor has a higher valuation of the firm than the manager does. Disagreement actually lowers the cost of capital in this state from the manager's perspective, and *ceteris paribus* the investor requires a lower ownership, α, for a lower value of the agreement parameter, ρ, if θ_N is high enough. That assumption rules out this possibility and guarantees that the manager does not prefer disagreement to agreement with the investor.

the manager's project choice. This permits the manager to retain complete control without any impact on the cost of capital. That is, with riskless debt financing, the manager is able to *simultaneously* achieve the highest possible valuation of the firm (since she has complete control) and the lowest possible cost of capital (a repayment of \$1).

Proposition 3. *If the value of the AIP is $F \geq 1$ for sure, then the optimal security design is riskless debt with exclusive managerial control.*

B. The Value of the AIP Is $\mathbf{F} \in (\mathbf{0}, \mathbf{1})$

Since the value of the AIP is smaller than \$1 in this case, a debt contract that gives the investor a priority claim over the AIP with exclusive managerial control cannot ensure the investor a repayment of \$1 in every state. However, optimal security design can be achieved via a debt-equity mix. Since there is no disagreement regarding the value of the AIP, the manager can first raise $\$F$ financing by issuing a riskless debt with promised repayment $\$F$ which gives the investor a priority claim over the AIP. Thus, the only cash flow remaining in the firm at $t = 2$ after the AIP is used to repay the debtholder is the project cash flow. We know from Proposition 1 that when the manager and the investor are sharing only the project cash flow, the optimal security design is equity. Thus, the manager should raise the remaining $\$[1 - F]$ through equity financing.

Proposition 4. *If the value of the AIP is $F \in (0, 1)$ for sure, then the optimal security design is a debt-equity mix that raises $\$F$ from debt and $\$[1 - F]$ from equity. The debt contract stipulates a repayment of $\$F$, has a priority claim over the AIP, and investors who purchase this security provide exclusive control to the manager. For the equity contract, if $\sqrt{[1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]} \in (B_1 + B_2, B_1)$, then the optimal managerial autonomy parameter is:*

$$\eta^* = \frac{\sqrt{[1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]} - B_1}{B_2} \in (0, 1). \tag{21}$$

The corresponding ownership share sold to the equity investor is given by:

$$\alpha^* = \sqrt{\frac{1-F}{B_1 + B_3[1-\gamma H]/[H-1]}}. \tag{22}$$

If $\sqrt{[1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]} \leq [B_1 + B_2]$, then the optimal managerial autonomy parameter is $\eta^* = 1$. The corresponding ownership share sold to the equity investor is given by:

$$\alpha^* = \frac{1-F}{B_1 + B_2}. \tag{23}$$

If $\sqrt{[1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]} \geq B_1$*, then the optimal managerial autonomy parameter is* $\eta^* = 0$*. The corresponding ownership share sold to the equity investor is given by:*

$$\alpha^* = \frac{1-F}{B_1}. \tag{24}$$

The idea of designing this debt-equity mix is as follows. Since there is no disagreement between the manager and the investor regarding the value of the AIP, an optimal security design should fully utilize the AIP to raise the \$1 financing. If $F \geq 1$, the AIP alone enables the manager to retain complete control via issuing riskless debt that has a priority claim over the AIP and provides sufficient financing on its own to cover the entire project investment. This riskless debt contract is the first-best solution to the optimal security design problem. If $F < 1$, the first-best is no longer attainable. However, a debt-equity mix enables the manager to fully utilize the AIP to minize the cost associated with disagreement on the one hand, and fully exploit the optimality of equity in dealing with project-cash-flow disagreement on the other hand. Note that this debt-equity mix strictly dominates any asset-backed securitization program (e.g., ABCP) in this case, since the debt-equity mix essentially fulfills the same function of such securitization program without even segregating the AIP from the rest of the firm, which is costly.

Denote the manager's expected payoff as $\pi(F,\rho)$, i.e., $\pi(F,\rho) \equiv [1-\alpha^*]\mathbf{E}_m(Z)|_{\{\eta=\eta^*\}}$, where α^* and η^* are specified in Proposition 4. Note that $\pi(F,\rho)$ is only defined for $F < 1$. The following lemma establishes the properties of $\pi(F,\rho)$.

Lemma 3. $\pi(F,\rho)$ *is increasing in* ρ*. It is also a convex and increasing function of* F*.*

The intuition for the result that $\pi(F,\rho)$ is increasing in ρ is that a higher agreement parameter reduces the cost associated with disagreement regarding the project cash flow and hence lowers the cost of capital. An increase in F has two consequences. First, it enables the manager to shift more financing from equity (with disagreement risk) to riskless debt (without disagreement risk). This is why $\pi(F,\rho)$ is increasing in F. Second, as the amount of equity financing decreases as F increases, disagreement becomes *increasingly* less important to the investor (because the stake that is at risk shrinks). Consequently, $\pi(F,\rho)$ is a convex function of F.

4.3 Stochastic Value of the AIP

We now study the case in which the value of the AIP is stochastic ex ante at $t = 0$, $\tilde{F}$, with probability density $g(\cdot)$ and cumulative distribution function $G(\cdot)$. Clearly, if supp $g(\cdot) =$

$[1, \infty)$, we know from Proposition 3 that riskless debt with a priority claim over the AIP and exclusive managerial control is the optimal security design. Thus, we focus now on the cases in which supp $g(\cdot) = [0, \infty)$ and supp $g(\cdot) = [0, 1)$. Denote $\bar{F} \equiv \mathbf{E}(\tilde{F})$ as the expected value of the AIP at $t = 0$. We distinguish three cases: (i) supp $g(\cdot) = [0, \infty)$ with $\bar{F} \geq 1$; (ii) supp $g(\cdot) = [0, \infty)$ with $\bar{F} < 1$; and (iii) supp $g(\cdot) = [0, 1)$.

Ideally, if the firm can segregate its AIP from the project cash flow and issue a debt claim against the AIP only, the manager can minimize the cost associated with disagreement regarding the project cash flow. For example, in case (i) indicated above, if the manager can write a contract against the AIP only, she can issue a risky debt claim with its repayment obligation set such that the expected payoff for the debtholder is \$1. Hence, the debtholder is willing to extend \$1 in funding to the firm. Moreover, since the debtholder's claim is limited to the AIP, he is indifferent to the manager's choice of project and is thus willing to provide complete project-choice autonomy to the manager. This contract eliminates all disagreement risk and is thus a first-best solution. Segregating the firm's AIP from the rest of the firm involves a fixed cost S, according to our model setup. We interpret such segregation as the creation of Asset Backed Commercial Paper (ABCP).

An alternative to segregating the AIP from the project cash flow is to issue convertible debt ex ante at $t = 0$ to implement the ex post optimal strategy as implied by Proposition 4 when the value of the AIP is realized at $t = 0.5$. The convertible debt contract stipulates that when the realized value of the AIP is $F < 1$, the investor converts $\$[1 - F]$ of the debt into equity (with managerial autonomy and ownership parameters sepecified in Proposition 4 for each possible realized value of the AIP) and keeps the remaining $\$F$ as debt; when the realized value of the AIP is $F \geq 1$, no conversion occurs and the manager achieves complete project-choice autonomy with the lowest cost of capital. The following analysis compares the two choices for the three different cases.

A. supp $\mathbf{g}(\cdot) = [\mathbf{0}, \infty)$ *with* $\bar{\mathbf{F}} \geq \mathbf{1}$

If the manager issues ABCP with a fixed cost S, she is able to retain complete control over project choice and achieve the lowest cost of capital. Thus, the manager's expected payoff (denoted as Π_{ABCP}) is:

$$\begin{aligned} \Pi_{ABCP} &= \mathbf{E}\left(1 + \gamma\theta_P[H-1] + \tilde{F}\right) - 1 - S \\ &= \bar{F} + \gamma\theta_P[H-1] - S. \end{aligned} \tag{25}$$

If the manager issues covertible debt, she faces two situations ex post at $t = 0.5$: (i) if the realized value of the AIP is $F \geq 1$, no conversion occurs and she is able to achieve complete control with the lowest cost of capital; and (ii) if the realized value of the AIP is $F < 1$, then the investor converts $\$[1 - F]$ of the debt into equity with managerial autonomy parameter (η^*) and ownership parameter (α^*) specified in Proposition 4. Thus, the manager's expected payoff (denoted as Π_{CON}) is:

$$\Pi_{CON} = [1 - G(1)]\mathbf{E}\left(\tilde{F} + \gamma\theta_P[H-1]|\tilde{F} \geq 1\right) + G(1)\mathbf{E}\left(\pi(\tilde{F}, \rho)|\tilde{F} < 1\right). \quad (26)$$

It is clear that when $\rho = 1$, there is no disagreement risk associated with equity financing and we have $\Pi_{CON} = \left[\bar{F} + \gamma\theta_P[H-1]\right] > \Pi_{ABCP}$.

B. supp $\mathbf{g}(\cdot) = [\mathbf{0}, \infty)$ ***with*** $\bar{\mathbf{F}} < \mathbf{1}$

If the manager issues ABCP, she is able to raise $\$\bar{F}$ backed by the AIP only. The remaining $\$[1 - \bar{F}]$ is financed through equity. Thus, the manager's expected payoff is:

$$\Pi_{ABCP} = \pi(\bar{F}, \rho) - S. \quad (27)$$

If the manager issues convertible debt, her expected payoff is given by (26).

C. supp $\mathbf{g}(\cdot) = [\mathbf{0}, \mathbf{1})$

If the manager issues ABCP, her expected payoff is given by (27). If the manager issues convertible debt, her expected payoff is given by:

$$\Pi_{CON} = \mathbf{E}\left(\pi(\tilde{F}, \rho)\right). \quad (28)$$

The following proposition summarizes the above discussion for the case in which the value of the AIP is stochastic.

Proposition 5. *When the value of the AIP is stochastic, the optimal security design depends on the distribution of $\tilde{F}$ in the following way:*

1. *When supp $g(\cdot) = [1, \infty)$, riskless debt with exclusive managerial control is the optimal security design.*

2. *When supp $g(\cdot) = [0, \infty)$ and $\bar{F} \geq 1$, there are two cases: (a) if*

$$S \leq G(1)\left[\gamma\theta_P[H-1] - \mathbf{E}\left(\pi(\tilde{F}, 0)|\tilde{F} < 1\right)\right] + \bar{F} - [1 - G(1)]\mathbf{E}\left(\tilde{F}|\tilde{F} \geq 1\right),$$

there exists a cutoff ρ_2^*, such that for $\rho \geq \rho_2^*$ the manager finds it optimal to issue convertible debt, whereas for $\rho < \rho_2^*$ the manager finds it optimal to issue ABCP; and (b) if

$$S > G(1)\left[\gamma\theta_P[H-1] - \mathbf{E}\left(\pi(\tilde{F}, 0)|\tilde{F} < 1\right)\right] + \bar{F} - [1 - G(1)]\mathbf{E}\left(\tilde{F}|\tilde{F} \geq 1\right),$$

the manager always finds it optimal to issue convertible debt.

3. *When supp $g(\cdot) = [0, \infty)$ and $\bar{F} < 1$, there are two cases: (a) if*

$$S \leq \pi(\bar{F}, \rho) - [1 - G(1)]\Big[\gamma\theta_P[H-1] + \mathbf{E}(\tilde{F}|\tilde{F} \geq 1)\Big] - G(1)\mathbf{E}\left(\pi(\tilde{F}, 0)|\tilde{F} < 1\right),$$

there exists a cutoff ρ_1^*, such that for $\rho \geq \rho_1^*$ the manager finds it optimal to issue convertible debt, whereas for $\rho < \rho_1^*$ the manager finds it optimal to issue ABCP; and (b) if

$$S > \pi(\bar{F}, \rho) - [1 - G(1)]\Big[\gamma\theta_P[H-1] + \mathbf{E}(\tilde{F}|\tilde{F} \geq 1)\Big] - G(1)\mathbf{E}\left(\pi(\tilde{F}, 0)|\tilde{F} < 1\right),$$

the manager always finds it opitmal to issue convertible debt.

4. *When supp $g(\cdot) = [0, 1)$, the optimal security design is convertible debt, in which the investors convert $\$[1 - F]$ of the debt into equity with pre-specified managerial autonomy and ownership parameters as in Proposition 4.*

The intuition is as follows.[18] When supp $g(\cdot) = [0, 1)$, we have $\bar{F} < 1$. Neither ABCP nor convertible debt can avoid disagreement risk both ex ante (at $t = 0$ before the value of the AIP is realized) and ex post (at $t = 0.5$ after the value of the AIP is realized). When the (ex post) realized value of the AIP is smaller than $\bar{F}$, ABCP limits the manager's loss due to disagreement while convertible debt does not offer such protection; when the realized value of the AIP is greater than $\bar{F}$, convertible debt enables the manager to implement an ex post optimal contracting strategy while ABCP cannot. Since the manager's payoff function $\pi(F, \rho)$ is convex in F, the upside benefit dominates the downside loss, and hence convertible debt is an ex ante optimal security design. However, this intuition does not extend to the case in which supp $g(\cdot) = [0, \infty)$. This is because $\pi(F, \rho)$ is only defined for $F < 1$: when $F \geq 1$, the manager's payoff is a linear, but not convex, function of F. Here the choice between ABCP and convertible debt depends on the value of the agreement parameter. When supp $g(\cdot) = [0, \infty)$ with $\bar{F} \geq 1$, ABCP always yields the manager the highest possible valuation of the firm and the lowest possible cost of capital, although she must pay a fixed cost S to obtain this. However, convertible debt is always costly ex ante because of the possibility that part of the financing will be done through equity ex post. The higher is the agreement parameter, the less costly is the ex ante cost of equity financing. Thus, if S is small enough, ABCP dominates convertible debt when ρ is low, whereas convertible debt dominates ABCP when ρ is high. If S is sufficiently large, the fixed cost associated with ABCP is such that the manager finds it inefficient to issue ABCP. Similar intuiton applies to the case in which supp $g(\cdot) = [0, \infty)$ with $\bar{F} < 1$.

[18] The reason why we separate the case in which supp $g(\cdot) = [0, \infty)$ and $\bar{F} \geq 1$ from the case in which supp $g(\cdot) = [0, \infty)$ and $\bar{F} < 1$ is to clarify that while both the cutoffs, ρ_1^* and ρ_2^*, depend on the value of F, each depends on F in a different way.

One implication of this analysis is that convertible debt use will tend to dominate ABCP when ρ, and hence the firm's stock price, is relatively high, and ABCP will tend to be preferred when the firm's stock price is relatively low. This follows from the result that the investor's valuation of the firm is increasing in ρ (see Lemma 2).

In our next result, we establish that there are exogenous parameters such that all the securities we have examined could arise as optimal securities.

Proposition 6. *There exists a non-empty set of exogenous parameter values such that we can obtain equity with joint control, equity with exclusive control, riskless debt, risky debt, convertible debt and ABCP as optimal security designs depending on the values of these exogenous parameters.*

This proposition says that the parametric restrictions we have imposed on the model are not overly restrictive.

5 MODEL EXTENSIONS AND EMPIRICAL IMPLICATIONS

In this section, we extend our base model to take up two issues. First, we prove the optimality of the linear sharing rule associated with equity in a more general setting than our three-state cash flow model. Second, we study the timing of equity issuance.

5.1 The Optimality of a Linear Sharing Rule for the General Case

In the base model, we established the optimality of equity when there are three possible values for the project cash flow, i.e., $Z \in \{0, 1, H\}$. We show in this section that the optimality of a linear sharing rule persists even when there are more than three possible values for the project cash flow.

We assume that the commonly-shared prior beliefs at $t = 0$ are that a good (G) project's payoff at $t = 2$ can take Ω possible values, $\{H_1, \ldots, H_\Omega\}$, with $H_i < H_j$ for $\forall i < j$, and $H_1 < 1$ and $H_\Omega > 1$. The probability that a good project realizes a payoff of H_j is $f_j \in [0, 1]$, with $\sum_{j=1}^{\Omega} f_j = 1$. Hence, the expected payoff for a good project is $H_G \equiv \sum_{j=1}^{\Omega} H_j f_j$. Similarly, a bad ($B$) project's payoff at $t = 2$ can also take Ω possible values, $\{H_1, \ldots, H_\Omega\}$. The probability that a bad project realizes a payoff of H_j is $b_j \in [0, 1]$, with $\sum_{j=1}^{\Omega} b_j = 1$. The expected payoff for a bad project is $H_B \equiv \sum_{j=1}^{\Omega} H_j b_j$. We assume $H_G > 1 > H_B$.[19] We also assume that

[19] We can easily generalize the analysis to continuously distributed project payoffs and show that the result still holds.

$\gamma H_G + [1-\gamma]H_B < 1$ and $\hat{\gamma}H_G + [1-\hat{\gamma}]H_B = 1$, i.e., a priori the project has negative NPV, and after seeing a not-precise, good signal ($s = s_G, q = N$) the agent perceives the project to have zero NPV.

In this general case, the project cash flow, Z, can take Ω possible values if $H_j = 1$ for some j; otherwise, Z can take $[\Omega+1]$ possible values.[20] The analyses for the two cases are the same, so we focus on the latter case in which Z can take $[\Omega+1]$ possible values, i.e., $Z \in \{1, H_1, \ldots, H_\Omega\}$. Note that $Z = 1$ corresponds to no investment, and $1 = \hat{\gamma}H_G + [1-\hat{\gamma}]H_B = \hat{\gamma}\big[H_G + [1-\hat{\gamma}]/\hat{\gamma}H_B\big]$. Define $\bar{H} \equiv H_G + [1-\hat{\gamma}]/\hat{\gamma}H_B$.

Consider a general sharing rule such that the investor gets α_j share of the project cash flow when $Z = H_j$ and α share of the project cash flow when $Z = 1$. We want to demonstrate that the optimal sharing rule is such that $\alpha_j = \alpha$ for $\forall j \in \{1, \ldots, \Omega\}$. Note that $\sum_{j=1}^{\Omega} \alpha_j H_j[\hat{\gamma}f_j + [1-\hat{\gamma}]b_j] = [\bar{\alpha}/\hat{\gamma}]\sum_{j=1}^{\Omega} H_j[\hat{\gamma}f_j + [1-\hat{\gamma}]b_j] = [\bar{\alpha}/\hat{\gamma}]\big[\hat{\gamma}H_G + [1-\hat{\gamma}]H_B\big] = \bar{\alpha}\bar{H}$ for some $\bar{\alpha}$. We can always find such an $\bar{\alpha}$. Thus, we can sort of "aggregate" all the possible values for the project cash flow and show that these are equivalent to having one cash flow for G (i.e., $\bar{H}$) and one cash flow for B (i.e., 0). That is, there is always an equivalence between the contract in this general case in which there are multiple (more than three) possible values for the project cash flow and the contract in the base model in which there are only three possible values for the project cash flow. For a linear contract, we have $\bar{\alpha} = \alpha$. For any general contract that is not globally linear (i.e., as long as there exists a j such that $\alpha_j \neq \alpha$), we may have $\bar{\alpha} \neq \alpha$. If $\bar{\alpha} > \alpha$, we are back to the base model with a convex contract, which cannot be optimal; if $\bar{\alpha} < \alpha$, we are back to the base model with a concave contract, which again is not optimal. In general, the optimal linear contract can not be dominated.

5.2 Timing of Equity Issuance

Our previous analysis has suppressed managerial effort, e (introduced in Section 3.3), since e affects all forms of security design in the same way. However, to analyze the timing of the manager's equity issuance, it is important that we explicitly take e into account. We know from Lemma 2 that the manager's net valuation of the firm, denoted as $\pi_m(\rho) \equiv [1-1/\mathbf{E}_i(Z)]\mathbf{E}_m(Z)$, is increasing in the agreement parameter ρ. The manager's expected utility with a choice of $e = 1$, net of her effort disutility cost, $e\zeta$, is then $U_m(\rho, \zeta) = \pi_m(\rho) - \zeta$. Assume that $\zeta \in (\pi_m(0), \pi_m(1)) \cap (\gamma H, 1)$; recall that $\zeta \in (\gamma H, 1)$ is sufficient to ensure that the investor will not bribe the manager to take the project when the manager strictly prefers to reject it.

[20] Recall that if the project is not invested, $Z = 1$. Thus, if $H_j = 1$ for some j, Z can take Ω possible values: $Z \in \{H_1, \ldots, H_\Omega\}$; otherwise, Z can take $[\Omega + 1]$ possible values: $Z \in \{1, H_1, \ldots, H_\Omega\}$.

Since the manager must choose $e = 1$ in equilibrium, $U_m(\rho, \zeta)$ is the manager's equilibrium expected utility. To satisfy the manager's participation constraint, we must have $U_m(\rho, \zeta) \geq 0$. Since $\partial U_m(\rho, \zeta)/\partial \rho > 0$, it follows that there exists a cutoff $\rho^* \in (0, 1)$, such that $U(\rho^*, \zeta) = 0$, $U_m(\rho, \zeta) > 0$ for $\forall \rho > \rho^*$, and $U_m(\rho, \zeta) < 0$ for $\forall \rho < \rho^*$. Thus, the manager issues equity only when $\rho \geq \rho^*$. The reason is that the manager will find it worthwhile to raise external financing only if she perceives a sufficiently high probability of being able to invest in the project. This probability is increasing in the agreement parameter ρ. Combining the observation that equity is issued only when $\rho \geq \rho^*$ with Lemma 2 yields the following:

Proposition 7. *The manager issues equity only when the firm's stock price is sufficiently high.*

The empirical observation that firms tend to issue equity only when their stock prices are sufficiently high is well known by now. But we lack a coherent explanation for why, particularly in light of the fact that a (static) optimal capital structure in market value terms would predict the exact opposite, namely that firms should issue debt during periods of high stock prices in order to return to a target capital structure. Our analysis indicates that the empirically-documented behavior of firms is a natural consequence of the emergence of equity as an optimal security design.

5.3 Empirical Implications

Our analysis has numerous empirical implications. We discuss them below. Any empirical test will need to confront the issue of appropriate proxies for the agreement parameter ρ and the autonomy parameter η. We believe one useful proxy for ρ is the announcement effect associated with an acquisition, since these effects are unlikely to be induced by private information the acquirer has about the target. The higher is ρ, the more positive will be the acquisition announcement effect. Another proxy may be the number of proxy fights, with fewer proxy fights denoting a higher ρ. A good proxy for η may be derived from charter provisions in initial public offerings.

1. Proposition 3 states that if the firm's assets in place have a sufficiently high known value, debt financing is preferred, whereas Proposition 4 states that if the assets in place have a relatively small known value, equity and debt will both be used. Moreover, Proposition 4 implies that the debt-equity ratio will be increasing in the value of the firm's assets in place. Thus, the prediction is that firms with larger tangible assets compared to the value of future investment opportunities will have higher levels of debt in their capital structures even in the absence of taxes. If book value represents the value of assets in

place, whereas market value also reflects the value of future investment opportunities, then our analysis predicts that firms with lower market-to-book ratios will issue more debt and firms with higher market-to-book ratios will issue more equity. Evidence consistent with this prediction can found in Asquith and Mullins [1986], Jung, Kim and Stulz [1996], Marsh [1982], and Mikkelson and Partch [1986].

2. Propositions 3 and 4 also imply that there is a life-cycle effect in the firm's financing choice. For start-up firms with scant assets in place, equity financing is optimal. However, as the firm's assets in place grow, it will eventually switch to debt financing. A switch back to equity will occur when the firm's external financing needs can no longer be secured by only its assets in place and when ρ is sufficiently high, as reflected in a high stock price. The empirical prediction is that firms start out being equity financed (possibly private equity), use debt financing when their assets in place are relatively high, and then issue equity again when they grow larger and their stock prices are high.

3. From Proposition 5, we get the prediction that when the value of the firm's assets in place is such that no riskless debt can be issued, the firm will issue either convertibles or ABCP. Convertible debt is issued when the cost of setting up an ABCP program is sufficiently high or when the firm's stock price is relatively high. Otherwise, ABCP will be issued.[21]

4. From Proposition 7, we also know that the firm is more likely to issue equity when its stock price is higher. This is consistent with the empirical evidence in Baker and Wurgler [2002], Graham and Harvey [2001], and Welch [2004].

6 CONCLUSION

We have examined optimal security design from the perspective of efficiently allocating cash-flow sharing rights and control rights. The allocation of control rights becomes relevant *not* because of agency or asymmetric information problems but due to fundamental disagreement over project choice arising from heterogeneous, rational beliefs about the value of the project. With this approach, cash-flow sharing rights and control rights are two side of the same coin and must be jointly determined in any optimal security design.

Our analysis shows the optimality of debt, equity, convertible debt and asset-backed commercial paper under different circumstances. Relative to the existing literature on the optimal-

[21] The assets backing up the commercial paper issue in an ABCP program are typically inventories and receivables, i.e., assets whose value there should not be much disagreement about, as in our analysis.

ity of debt, our marginal contributions are to show that debt may be optimal even when the firm's cash flow is costlessly observable ex post, and to distinguish the circumstances under which straight debt is used and those under which asset-backed commercial paper is used. Relative to the existing literature on equity, our marginal contribution is to demonstrate the optimality of joint control between the owner-manager and outside shareholders. Relative to the existing literature on convertible debt, our marginal contribution is to show that convertible debt is the optimal security design when the firm's assets in place have relatively high uncertainty and the firm's stock price is relatively high, and that the optimality of convertible debt may be encountered even in the absence of agency or asymmetric information problems. Moreover, ours is the first paper to show that with sufficiently uncertain assets in place, the firm's choice boils down to one between convertible debt and asset-backed commercial paper and that the latter is efficient to use when the stock price is relatively low. An important aspect of our analysis is that the efficient cash-flow sharing rule and the efficient control rights allocation rule are intertwined and cannot be determined in isolation of each other. In particular, the choice between debt and equity is often a choice between exclusive control and joint control; exclusive control is encountered whenever debt is the efficient choice, whereas equity is the efficient choice when joint control is optimal.

In addition to these main results, the analysis provides a security design perspective on why the very design of equity and convertible debt as optimal securities dictates that firms will find it efficient to issue these when their stock prices are high. We believe that managerial autonomy considerations may prove useful in also understanding various other aspects of security design, such as preferred stock, non-voting equity and debt maturity.[22] These await further research.

[22]For an empirical analysis of dual-class shares, which include both voting and non-voting shares, see Gompers, Ishii and Metrick [2003].

Appendix

Proof of Proposition 1: We first show that a convex sharing rule cannot be optimal, then we demonstrate that under Assumption 2, a concave sharing rule cannot be optimal either.

Convex Sharing Rule Cannot Be Optimal: Note that the convex sharing rule needs not be strictly convex because we have only three possible values for Z. That is, by saying convex sharing rule, we are essentially referring a sharing rule in which $\alpha_H > \alpha_1$ while the value of α_0 is irrelevant. We demonstrate that any convex sharing rule is dominated by a linear sharing rule.

First, suppose that the sharing rule $\Phi(\cdot)$ is not convex enough so that the investor still blocks the manager in the state $\{s = s_G, q_m = P, q_i = U\}$ with some probability. Note that the investor never blocks the manager in the state $\{s = s_G, q_m = P, q_i = N\}$ under a convex sharing rule, since if he blocks the manager he gets α_1 whereas if he does not block the manger he gets $\hat{\gamma}[\alpha_H H] = \alpha_H > \alpha_1$. Formally, assume that some specific $\alpha_H > \alpha_1$, α_0 and η constitute an optimal convex sharing rule. Note that the investor's participation constraint is binding at equilibrium:

$$\begin{aligned}\mathbf{E}_i(\Phi(Z)) &= \gamma\theta_P\rho[\alpha_H H] + \gamma\theta_P[1-\rho]\tau\left[\hat{\gamma}[\alpha_H H]\right] + \gamma\theta_P[1-\rho][1-\tau][\eta]\left[\gamma[\alpha_H H]\right] \\ &\quad + \gamma\theta_P[1-\rho][1-\tau][1-\eta][\alpha_1] + [1-\gamma\theta_P][\alpha_1] \\ &= 1. \end{aligned} \tag{A1}$$

Note that the manager does not invest in the state $\{s = s_G, q_m = N, q_i = P\}$. The manager perceives the cost of capital to be:

$$\begin{aligned}\mathbf{E}_m(\Phi(Z)) &= \gamma\theta_P\rho[\alpha_H H] + \gamma\theta_P[1-\rho]\tau\left[\alpha_H H\right] + \gamma\theta_P[1-\rho][1-\tau][\eta]\left[\alpha_H H\right] \\ &\quad + \gamma\theta_P[1-\rho][1-\tau][1-\eta][\alpha_1] + [1-\gamma\theta_P][\alpha_1] \\ &= 1 + \gamma\theta_P[1-\rho]\left[\tau\left[1-\hat{\gamma}\right] + \eta[1-\tau][1-\gamma]\right][\alpha_H H], \end{aligned} \tag{A2}$$

and the value of the firm as:

$$\begin{aligned}\mathbf{E}_m(Z) &= \gamma\theta_P\rho[H] + \gamma\theta_P[1-\rho]\tau\left[H\right] + \gamma\theta_P[1-\rho][1-\tau][\eta]\left[H\right] \\ &\quad + \gamma\theta_P[1-\rho][1-\tau][1-\eta][1] + [1-\gamma\theta_P][1]. \end{aligned} \tag{A3}$$

Note that the cost of capital $\mathbf{E}_m(\Phi(Z))$ is increasing in α_H, whereas the value of the firm $\mathbf{E}_m(Z)$ solely depends on η. Let us keep η fixed, decrease α_H by Δ_H, and increase α_1 by Δ_1 to keep the investor's participation constraint binding ($\mathbf{E}_i(\Phi(Z)) = 1$). By doing this, the value of the firm is unchanged (since η is unchanged), while the cost of capital is decreased (since α_H is decreased). The manager is better off. Δ_1 can be determined as (to keep $\mathbf{E}_i(\Phi(Z)) = 1$):

$$\Delta_1 = \Delta_H\left[\frac{\gamma\theta_P\left[\rho + [1-\rho]\tau\hat{\gamma} + [1-\rho][1-\tau]\eta\gamma\right]H}{1-\gamma\theta_P + \gamma\theta_P[1-\rho][1-\tau][1-\eta]}\right] > 0. \tag{A4}$$

We can continue adjusting the convex sharing rule by decreasing α_H and increasing α_1 to improve the manager's expected payoff. The limit of the adjustment is a linear sharing rule.

Second, if the convexity is strong enough so that the investor does not block the manager in the state $\{s = s_G, q_m = P, q_i = U\}$, i.e., $\alpha_H[\gamma H] \geq \alpha_1$, we show that this cannot be an optimal sharing rule, either. Note in this case, we have:

$$\begin{aligned}\mathbf{E}_i(\Phi(Z)) &= \gamma\theta_P\rho[\alpha_H H] + \gamma\theta_P[1-\rho]\tau[\alpha_H] + \gamma\theta_P[1-\rho][1-\tau]\big[\gamma[\alpha_H H]\big] + [1-\gamma\theta_P][\alpha_1] \\ &= 1. \end{aligned} \tag{A5}$$

$$\begin{aligned}\mathbf{E}_m(\Phi(Z)) &= \gamma\theta_P\rho[\alpha_H H] + \gamma\theta_P[1-\rho]\tau[\alpha_H H] + \gamma\theta_P[1-\rho][1-\tau][\alpha_H H] + [1-\gamma\theta_P][\alpha_1] \\ &= 1 + \gamma\theta_P[1-\rho]\big[\tau[H-1] + H[1-\tau][1-\gamma]\big][\alpha_H], \end{aligned} \tag{A6}$$

and

$$\mathbf{E}_m(Z) = 1 + \gamma\theta_P[H-1]. \tag{A7}$$

Note again that the value of the firm $\mathbf{E}_m(Z)$ does not depend on the sharing rule, whereas the cost of capital $\mathbf{E}_m(\Phi(Z))$ is strictly decreasing in α_H. Thus, we must have $\alpha_H[\gamma H] = \alpha_1$ at optimum in this case, i.e., the convexity is strong enough but not too strong so that the investor is just indifferent between blocking and not blocking the manger in the state $\{s = s_G, q_m = P, q_i = U\}$. However, we show that this cannot be an optimum. We demonstrate as follows.

Suppose the manager proposes to the investor: I will decrease α_H by a small amount Δ'_H, but if you do not block me in the state $\{s = s_G, q_m = P, q_i = U\}$ as before, in return I will increase α_1 by Δ'_1 to keep your participation constraint binding. It can be shown that

$$\Delta'_1 = \Delta'_H \left[\frac{\gamma\theta_P\big[\rho + [1-\rho]\tau\hat{\gamma} + [1-\rho][1-\tau]\gamma\big]H}{1-\gamma\theta_P}\right] > 0. \tag{A8}$$

By doing this, the cost of capital decreases, while the value of the firm is unchanged. Thus, the manager is better off. But by reducing the convexity in this manner, i.e., decreasing α_H and increasing α_1, at some point the manager will find that it is optimal to give up some autonomy instead of further giving up sharing of firm value (i.e., instead of increasing α_1 further). But then we are back to the first case in which we have shown that a linear cash-flow sharing rule dominates.

Concave Sharing Rule Cannot Be Optimal: Suppose that $\alpha_H < \alpha_1$, α_0 and η constitute an optimal concave sharing rule. We now demonstrate that by reducing the concavity, the manager's payoff can be improved under Assumption 2. Under a concave sharing rule, the investor strictly prefers to block the manager in the state $\{s = s_G, q_m = P, q_i = N\}$. To see this, note that if the investor blocks the manager, he gets α_1 whereas if he does not, he gets $\alpha_H[\hat{\gamma}H] = \alpha_H < \alpha_1$.

Before we move on, we first claim that the managerial autonomy parameter for an optimal concave contract can not be higher than a threshhold value, $\bar{\eta}$, which is the optimal managerial autonomy parameter when we set $\alpha_1 = 1$ (we solve for $\bar{\eta}$ below). It is assumed that $\bar{\eta}$ has the following property (from Assumption 2):

$$[1-\hat{\gamma}]\theta_N\hat{\tau} \leq \bar{\eta}\big[[1-\gamma][1-\tau] + [1-\hat{\gamma}]\tau\big]\theta_P. \tag{A9}$$

Multiplying both sides by $\alpha_H H$, we have

$$\alpha_H[H-1]\theta_N\hat{\tau} \leq \bar{\eta}[\alpha_H]\big[[H-\gamma H][1-\tau] + [H-1]\tau\big]\theta_P, \tag{A10}$$

where the left-hand-side (LHS) is the "bonus" to the manager in the state $\{s = s_G, q_m = N, q_i = P\}$ and the right-hand-side (RHS) is the cost to the manager due to disagreement in the states $\{s = s_G, q_m = P, q_i = N\}$ and $\{s = s_G, q_m = P, q_i = U\}$. We then claim that the managerial autonomy parameter for an optimal concave contract should not be higher than $\bar{\eta}$. To see this, note that for $\forall\eta > \bar{\eta}$, we have $[1-\hat{\gamma}]\theta_N\hat{\tau} < \eta\big[[1-\gamma][1-\tau] + [1-\hat{\gamma}]\tau\big]\theta_P$ (from (A9)). Thus, in an optimal concave contract with managerial autonomy parameter $\eta > \bar{\eta}$, the manager should set α_H as low as possible by setting $\alpha_1 = 1$ (from (A10)). But, conditional on $\alpha_1 = 1$, $\bar{\eta}$ is the optimal managerial autonomy parameter (based on the definition of $\bar{\eta}$). Hence, the managerial autonomy parameter in an optimal concave contract can not be greater than $\bar{\eta}$.

$\bar{\eta}$ is determined endogenously in the following manager's maximization problem:

$$\begin{aligned} &\max_{\{\alpha_1=1,\alpha_H,\bar{\eta}\}} \big[\mathbf{E}_m(Z) - \mathbf{E}_m(\Phi(Z))\big], \\ &\text{s.t. } \mathbf{E}_i(\Phi(Z)) = 1, \end{aligned} \tag{A11}$$

where

$$\begin{aligned} \mathbf{E}_m(Z) &= 1 + \gamma\theta_P\big[\rho + [1-\rho]\bar{\eta}\big][H-1], \\ \mathbf{E}_m(\Phi(Z)) &= 1 + \gamma[1-\rho]\big[[1-\hat{\gamma}][\theta_P\tau\bar{\eta} - \theta_N\hat{\tau}] + [1-\gamma]\theta_P[1-\tau]\bar{\eta}\big][\alpha_H H], \\ \mathbf{E}_i(\Phi(Z)) &= \gamma\theta_P[1-\rho][1-\bar{\eta}] + [1-\gamma\theta_P - \gamma\theta_N[1-\rho]\hat{\tau}] \\ &\quad + [\alpha_H]\big[\gamma\theta_P\rho H + \gamma\theta_P[1-\rho]\tau\bar{\eta} + \gamma\theta_P[1-\rho][1-\tau]\bar{\eta}[\gamma H] + \gamma\theta_N[1-\rho]\hat{\tau}H\big]. \end{aligned}$$

The solution to the problem yields the expression for $\bar{\eta}$ in Assumption 2.

Now consider a general concave sharing rule with $\eta \le \bar{\eta}$. Note that

$$\begin{aligned} \mathbf{E}_i(\Phi(Z)) &= \gamma\theta_P\rho[\alpha_H H] + \gamma\theta_P[1-\rho]\tau[\eta][\alpha_H] + \gamma\theta_P[1-\rho]\tau[1-\eta][\alpha_1] \\ &\quad + \gamma\theta_P[1-\rho][1-\tau][\eta]\big[\alpha_H[\gamma H]\big] + \gamma\theta_P[1-\rho][1-\tau][1-\eta][\alpha_1] \\ &\quad + \gamma\theta_N[1-\rho]\hat{\tau}[\alpha_H H] + \big[1 - \gamma\theta_P - \gamma\theta_N[1-\rho]\hat{\tau}\big][\alpha_1] \\ &= 1, \end{aligned} \tag{A12}$$

$$\mathbf{E}_m(\Phi(Z)) = 1 + \gamma[1-\rho][\alpha_H]\big[[\theta_P\tau\eta - \theta_N\hat{\tau}][H-1] + \theta_P[1-\tau]\eta[1-\gamma]H\big], \tag{A13}$$

and

$$\mathbf{E}_m(Z) = \gamma\theta_P\rho[H] + \gamma\theta_P[1-\rho][\eta][H] + \gamma\theta_P[1-\rho][1-\eta][1] + [1-\gamma\theta_P][1]. \tag{A14}$$

Suppose that we keep the managerial autonomy η unchanged, while at the same time adjust α_H upward to α'_H and adjust α_1 downward to α'_1 such that $\alpha'_H = \alpha'_1 \equiv \alpha \in (\alpha_H, \alpha_1)$. This is always possible. Thus, the investor will not block the manager in the state $\{s = s_G, q_m = P, q_i = N\}$. Denote $\Delta'' \equiv \alpha - \alpha_H > 0$ and $\alpha_1 - \alpha = \bar{\kappa}\Delta''$. It can be shown that

$$\bar{\kappa} = \frac{[\gamma H]\big[\theta_P\rho + \theta_N[1-\rho]\hat{\tau}\big] + \gamma\theta_P[1-\rho]\eta\big[\tau + [1-\tau][\gamma H]\big]}{1 - \gamma\big[\theta_P + \theta_N[1-\rho]\hat{\tau}\big] + \gamma\theta_P[1-\rho][1-\eta]} \ge \frac{[\gamma H]\big[\theta_P\rho + \theta_N[1-\rho]\hat{\tau}\big]}{1 - \gamma\big[\theta_P\rho + \theta_N[1-\rho]\hat{\tau}\big]}. \tag{A15}$$

Denote $\kappa \equiv \frac{[\gamma H]\big[\theta_P\rho + \theta_N[1-\rho]\hat{\tau}\big]}{1-\gamma\big[\theta_P\rho + \theta_N[1-\rho]\hat{\tau}\big]}$. Note that after the adjustment, the value of the firm increases by (this comes from the fact that the investor does not block the manager in the state $\{s = s_G, q_m = P, q_i = N\}$):

$$\gamma\theta_P[1-\rho]\tau[1-\eta][H-1]. \tag{A16}$$

But, the cost of capital $\mathbf{E}_m(\Phi(Z))$ also increases. We can show that $\mathbf{E}_m(\Phi(Z))$ increases by:

$$\gamma\theta_P[1-\rho][1-\tau][\eta][1-\gamma]H\Delta'' - \gamma\theta_N[1-\rho]\hat{\tau}[H-1]\Delta'' + \gamma\theta_P[1-\rho]\tau[H-1][\alpha-\alpha_H\eta], \qquad \text{(A17)}$$

where the first component $\gamma\theta_P[1-\rho][1-\tau][\eta][1-\gamma]H\Delta''$ represents the additional cost of capital to the manager because she increases α_H and hence the investor bears more loss (from the investor's perspective) in the state $\{s = s_G, q_m = P, q_i = U\}$; the second term $\gamma\theta_N[1-\rho]\hat{\tau}[H-1]\Delta''$ represents the additional bonus to the manager in the state $\{s = s_G, q_m = N, q_i = P\}$; the third term $\gamma\theta_P[1-\rho]\tau[H-1][\alpha-\alpha_H\eta]$ represents the additional cost of capital in the state $\{s = s_G, q_m = P, q_i = N\}$.

To break the optimality of the initial concave cash-flow sharing rule, all we need is that the increase in the value of the firm outweighs the increases in the cost of capital, i.e.,

$$\theta_N\hat{\tau}[H-1]\Delta'' \geq \theta_P\big[\tau\eta[H-1]\Delta'' + [1-\tau]\eta[H-\gamma H]\Delta'' - [1-\alpha]\tau[1-\eta][H-1]\big]. \qquad \text{(A18)}$$

It can be shown that (A18) is guaranteed by Assumption 2.[23] □

Proof of Lemma 1: We first derive $\mathbf{E}_i(Z)$ and $\mathbf{E}_m(Z)$. Note that the investor is indifferent between investment and non-investment when the state $\{s = s_G, q_m = P, q_i = N\}$ occurs. To see this, note that in this state if the investor blocks the manager, he gets $\hat{\gamma}[\alpha H] = \alpha$, and if he doesn't block the manager, he also gets $[\alpha \times 1] = \alpha$. We assume that the investor gives the manager full autonomy in this state. However, when the state $\{s = s_G, q_m = P, q_i = U\}$ occurs, the investor strongly prefers non-investment. Given the fact that the manager invests in the project with probability η in this state, the investor's valuation of the firm is:

$$\begin{aligned}\mathbf{E}_i(Z) \;&=\; \gamma\theta_P\rho H + \gamma\theta_P[1-\rho]\tau + \gamma\theta_P[1-\rho][1-\tau][\eta][\gamma H] + \gamma\theta_P[1-\rho][1-\tau][1-\eta] \\ &\quad + \gamma\theta_N[1-\rho]\hat{\tau}[H] + \big[1-\gamma\theta_P-\gamma\theta_N[1-\rho]\hat{\tau}\big], \end{aligned} \qquad \text{(A19)}$$

and the manager's valuation of the firm is:

$$\begin{aligned}\mathbf{E}_m(Z) \;&=\; \gamma\theta_P\rho H + \gamma\theta_P[1-\rho]\tau[H] + \gamma\theta_P[1-\rho][1-\tau][\eta][H] + \gamma\theta_P[1-\rho][1-\tau][1-\eta] \\ &\quad + \gamma\theta_N[1-\rho]\hat{\tau} + \big[1-\gamma\theta_P-\gamma\theta_N[1-\rho]\hat{\tau}\big]. \end{aligned} \qquad \text{(A20)}$$

Apply the notations for B_1, B_2 and B_3, we get the expressions for $\mathbf{E}_i(Z)$ and $\mathbf{E}_m(Z)$ in (13) and (14), respectively.

Note that $d\mathbf{E}_i(Z)/d\eta = B_2 < 0$. Since $\alpha = 1/\mathbf{E}_i(Z)$, we have $d\alpha/d\eta > 0$. For the second part of the lemma, note that for a fixed η, we have $d\big[[1-\alpha]\mathbf{E}_m(Z)\big]/d\alpha = -\mathbf{E}_m(Z) < 0$. For the last part of the lemma, note that for a fixed α, we have $d\big[[1-\alpha]\mathbf{E}_m(Z)\big]/d\eta = [1-\alpha]d\mathbf{E}_m(Z)/d\eta = [1-\alpha]B_2[H-1]/[\gamma H-1] > 0$. □

[23] Proof: Note the right-hand-side (RHS) of (A18) is increasing in η. Thus, a sufficiency condition to have (A18) is

$$\theta_N\hat{\tau}[H-1]\Delta'' \geq \theta_P\;\; \tau\bar{\eta}[H-1] + [1-\tau]\bar{\eta}[H-\gamma H] - \kappa\tau[1-\bar{\eta}][H-1]\;\; \Delta'',$$

which is guaranteed by Assumption 2.

Proof of Proposition 2: Define $\pi \equiv [1 - 1/\mathbf{E}_i(Z)]\,\mathbf{E}_m(Z)$. We first show that π is concave in η, i.e., $\pi_{\eta\eta} < 0$. Note

$$\pi_\eta = \left[1 - \frac{1}{\mathbf{E}_i(Z)}\right]\left[\frac{d\mathbf{E}_m(Z)}{d\eta}\right] + \left[\frac{\mathbf{E}_m(Z)}{[\mathbf{E}_i(Z)]^2}\right]\left[\frac{d\mathbf{E}_i(Z)}{d\eta}\right].$$

Note that both $d\mathbf{E}_m(Z)/d\eta$ and $d\mathbf{E}_i(Z)/d\eta$ are not functions of η. Thus,

$$\pi_{\eta\eta} = \frac{2}{[\mathbf{E}_i(Z)]^2}\left[\frac{d\mathbf{E}_i(Z)}{d\eta}\right]\left[\left[\frac{d\mathbf{E}_m(Z)}{d\eta}\right] - \left[\frac{d\mathbf{E}_i(Z)}{d\eta}\right]\left[\frac{\mathbf{E}_m(Z)}{\mathbf{E}_i(Z)}\right]\right].$$

Note that $d\mathbf{E}_i(Z)/d\eta < 0$ and $d\mathbf{E}_m(Z)/d\eta > 0$. Thus, we have $\pi_{\eta\eta} < 0$.

The first-order-condition (FOC) $\pi_\eta = 0$ can be explicitly written as:

$$B_2^2\eta^2 + 2B_1B_2\eta + \big[B_1^2 - B_1 + B_3[\gamma H - 1]/[H - 1]\big] = 0. \tag{A21}$$

There are two roots, $\eta_l < \eta_h$, for the equation. Note that

$$\eta_l + \eta_h = -\frac{2B_1}{B_2} > 2B_1 > 2.$$

Thus, we must have $\eta_h > 1$, which means that we should focus on η_l to search for η^*. To have an interior solution, we need the following assumptions: $\pi_\eta|_{\eta=0} > 0$ and $\pi_\eta|_{\eta=1} < 0$, i.e.,

$$\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]} \in (B_1 + B_2, B_1).$$

Solve the FOC, we get:

$$\eta^* = \frac{\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]} - B_1}{B_2} \in (0, 1). \tag{A22}$$

The investor's equity ownership α^* is determined as follows:

$$\alpha^* = \frac{1}{\mathbf{E}_i(Z)|_{\eta=\eta^*}} = \frac{1}{B_1 + B_2\eta^*} = \frac{1}{\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]}}. \tag{A23}$$

When $\pi_\eta|_{\eta=1} \geq 0$, i.e., $\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]} \leq [B_1 + B_2]$, we have $\eta^* = 1$, and the corresponding investor's equity ownership $\alpha^* = 1/[B_1 + B_2]$. When $\pi_\eta|_{\eta=0} \leq 0$, i.e., $\sqrt{B_1 + B_3[1 - \gamma H]/[H - 1]} \geq B_1$, we have $\eta^* = 0$, and the corresponding investor's equity ownership $\alpha^* = 1/B_1$. □

Proof of Lemma 2: We confine our analysis to the case with an interior solution for η^* because the comparative statics are meaningless at the two corner solutions. To show $d\alpha^*/d\rho < 0$, it is equivalent to show that $B_1 + B_3[1 - \gamma H]/[H - 1]$ is increasing in ρ. Note that

$$\begin{aligned}
\frac{d\big[B_1 + B_3[1 - \gamma H]/[H - 1]\big]}{d\rho} &\propto [\theta_P - \theta_N\hat{\tau}][H - 1] + \theta_P[1 - \tau][1 - \gamma H] \\
&= \big[[H - 1] + [1 - \tau][1 - \gamma H]\big]\theta_P - [H - 1]\theta_N\hat{\tau} \\
&\propto \big[1 + [1 - \tau][1 - \gamma H]/[H - 1]\big]\theta_P - \theta_N\hat{\tau} \\
&\propto \big[[\hat{\gamma} - \gamma][1 - \tau] + [1 - \hat{\gamma}]\big]\theta_P - [1 - \hat{\gamma}]\theta_N\hat{\tau} \\
&> 0,
\end{aligned} \tag{A24}$$

where the last inequality is guaranteed by Assumption 1.

Note the manager's net valuation of the firm is give by

$$\begin{aligned}[1-\alpha^*]\mathbf{E}_m(Z) &= [1-\alpha^*]\Big[B_3 + [H-1]/[\gamma H - 1]\big[1/\alpha^* - B_1\big]\Big] \\ &= [1/\alpha^* - 1]^2 \left[\frac{H-1}{1-\gamma H}\right]. \end{aligned} \tag{A25}$$

Since $d\alpha^*/d\rho < 0$, $[1-\alpha^*]\mathbf{E}_m(Z)$ is increasing in ρ. □

Proof of Proposition 3: Suppose $F \geq 1$ for sure. Then consider a security that gives the manager a share β of F and the investor a share $[1-\beta]$. If α_Z is the investor's share of the project cash flow when that cash flow is Z, then the manager solves

$$\max_{\{\beta,\Phi,\eta\}} \mathbf{E}_m\big(\beta F + [1-\alpha_Z]Z\big), \tag{A26}$$

$$\text{s.t. } \mathbf{E}_i\big([1-\beta]F + \alpha_Z Z\big) = 1. \tag{A27}$$

If we determine β as a solution to $[1-\beta]F = 1$, then $\beta = [F-1]/F$. With $\beta = [F-1]/F$, it is clear that (A27) is satisfied with $\alpha_Z = 0$ for $\forall Z$. Moreover, $\mathbf{E}_i([1-\beta]F) = 1$ is unaffected by η. Hence, it will be set at $\eta = 1$ because $\mathbf{E}_m(\cdot)$ is non-decreasing in η. The security thus provides the investor a priority claim on F and sets $\eta = 1$. Such a security yields the minimum cost of capital because the repayment obligation is minimized at \$1, and it yields the maximum managerial autonomy. Thus, it cannot be dominated. □

Proof of Proposition 4: We first prove the optimality of the debt-equity mix. As discussed in the text, we can rule out ABCP in the first place. Then any contract other than the debt-equity mix does not fully utilize the AIP. Consider an arbitrary contract that enables the investor to receive a share $[1-\beta]$ of F *on average* and a share α_Z of the project cash flow when that cash flow is Z. Thus, we have

$$[1-\beta]F + \mathbf{E}_i\left(\alpha_Z Z\right) = 1. \tag{A28}$$

We can always think of the financing is done via two parts with the first part raising $\$F$ and the second part raising $\$[1-F]$. In the debt-equity mix, the first part $\$F$ is completely backed by the AIP. This is no longer possible in this alternative contract, since only $[1-\beta]F$ is completely backed by the AIP. Thus, the manager needs to give the investor share of the project cash flow, say $\bar{\alpha}_Z$, to raise the first part financing (with $[1-\beta]F$ directly written on the AIP and the remaining βF from the project cash-flow), such that $\mathbf{E}_i(\bar{\alpha}_Z Z) = \beta F$. That is,

$$\underbrace{\big[[1-\beta]F + \mathbf{E}_i\left(\bar{\alpha}_Z Z\right)\big]}_{F} + \underbrace{\mathbf{E}_i\left([\alpha_Z - \bar{\alpha}_Z]Z\right)}_{[1-F]} = 1. \tag{A29}$$

However, note that $\mathbf{E}_m(\bar{\alpha}_Z Z) > \beta F$ and hence the cost of the first part financing is more than $\$F$ from the manager's perspective. For the second part $\$[1-F]$ financing, this alternative contract cannot dominate the equity identified in the debt-equity mix (from Proposition 1). Thus, the debt-equity mix is the optimal security design.

We now derive the optimal allocation of control rights for the equity contract in the debt-equity mix. The manager solves the following optimization problem:

$$\max_{\{\alpha,\eta\}} \mathbf{E}_m([1-\alpha]Z), \tag{A30}$$

$$\text{s.t. } \mathbf{E}_i(\alpha Z) = 1 - F. \tag{A31}$$

Following the proof of Proposition 2, the first-order-condition (FOC) is

$$[B_1 + B_2\eta]^2 = [1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]. \tag{A32}$$

Solving the FOC, we get the optimal allocation of control rights and cash-flow sharing. □

Proof of Lemma 3: We confine our analysis to the case in which there is an interior solution for η^*. We have $\pi(F,\rho) = [1-\alpha^*]\mathbf{E}_m(Z)$ with

$$\begin{aligned} [1-\alpha^*] &= 1 - \sqrt{\frac{1-F}{B_1 + B_3[1-\gamma H]/[H-1]}}, \\ \mathbf{E}_m(Z) &= B_3 - \left[\frac{H-1}{1-\gamma H}\right]\left[\sqrt{[1-F]\big[B_1 + B_3[1-\gamma H]/[H-1]\big]} - B_1\right]. \end{aligned}$$

It is clear that both $[1-\alpha^*]$ and $\mathbf{E}_m(Z)$ are convex increasing function of F. Thus, $\pi(F,\rho)$ is also a convex increasing function of F. The claim that $\pi(F,\rho)$ is increasing in ρ can be proved in the same manner as in the proof of Lemma 2. □

Proof of Proposition 5: If supp $g(\cdot) = [1,\infty)$, both the manager and the investor agree that the valuation of the firm is at least \$1 in every possible state at $t=2$. Thus, the manager is able to issue riskless debt with exclusive managerial control that achieves the first-best outcome.

If supp $g(\cdot) = [0,1)$, we have

$$\Pi_{CON} = \mathbf{E}\left(\pi(\tilde{F},\rho)\right) \geq \pi\left(\mathbf{E}(\tilde{F}),\rho\right) = \pi(\bar{F},\rho) > \pi(\bar{F},\rho) - S = \Pi_{ABCP},$$

since $\pi(F)$ is a convex function of F.

If supp $g(\cdot) = [0,\infty)$ and $\bar{F} < 1$, it is clear that when $\rho = 1$, we have $\Pi_{CON} > \Pi_{ABCP}$ since there is no disagreement cost associated with equity financing. If $S \leq \pi(\bar{F},\rho) - [1-G(1)]\Big[\gamma\theta_P[H-1] + \mathbf{E}(\tilde{F}|\tilde{F} \geq 1)\Big] - G(1)\mathbf{E}\left(\pi(\tilde{F},0)|\tilde{F} < 1\right)$, we have $\Pi_{CON} < \Pi_{ABCP}$ when $\rho = 0$. Since $\pi(F,\rho)$ is an increasing function of ρ, Π_{CON} is also increasing in ρ. Thus, the existence of the cutoff ρ_1^* is clear. If $S > \pi(\bar{F},\rho) - [1-G(1)]\Big[\gamma\theta_P[H-1] + \mathbf{E}(\tilde{F}|\tilde{F} \geq 1)\Big] - G(1)\mathbf{E}\left(\pi(\tilde{F},0)|\tilde{F} < 1\right)$, then $\Pi_{CON} > \Pi_{ABCP}$ when $\rho = 0$, and hence convertible debt dominates ABCP for $\forall\rho \in [0,1]$. The result for the case in which supp $g(\cdot) = [0,\infty)$ and $\bar{F} \geq 1$ can be proved similarly. □

Proof of Proposition 6: Define the set of exogenous parameter values $\Delta \equiv \{\theta_P, \theta_N, \theta_U, \hat{\gamma}, \gamma\}$. We prove Δ is non-empty by providing a numerical example. Let $\theta_P = 0.8, \theta_N = 0.1, \theta_U = 0.1, \hat{\gamma} = 0.6$ and $\gamma = 0.552$. These parameter values satisfy the two assumptions. We first vary the agreement parameter ρ to numerically illustrate Proposition 2 and Lemma 2, in which the value of the AIP is $F = 0$ for sure.

ρ	0.37	0.38	0.39	0.40	0.41	0.42	0.43	0.44
α^*	0.8861	0.8851	0.8841	0.8832	0.8822	0.8812	0.8803	0.8788
η^*	0.1842	0.2116	0.3432	0.4794	0.6203	0.7662	0.9174	1
$[1-\alpha^*]\mathbf{E}_m(Z)\|_{\{\eta=\eta^*\}}$	0.0124	0.0315	0.0515	0.0725	0.0946	0.1178	0.1422	0.1569

It is clear that as predicted by Lemma 2, when the agreement parameter ρ increases, the cost of capital α^* decreases, whereas the manager's net valuation of the firm $[1-\alpha^*]\mathbf{E}_m(Z)|_{\{\eta=\eta^*\}}$ increases.

We now illustrate Proposition 4 for the case in which $F \in (0,1)$. Let $F = 0.01$ and $\theta_P, \theta_N, \theta_U, \hat{\gamma}, \gamma$ take the same values as above, we have

ρ	0.37	0.38	0.39	0.40
α^*	0.8816	0.8806	0.8797	0.8786
η^*	0.5926	0.7287	0.8694	1
$\pi(F,\rho)$	0.0908	0.1126	0.1354	0.1571

Again, the cost of capital α^* is decreasing in ρ, whereas the manager's net valuation $\pi(F,\rho)$ is increasing in ρ.

We then show $\pi(F,\rho)$ is convex and increasing in F (Lemma 3). Let $\rho = 0.36$ and $\theta_P, \theta_N, \theta_U, \hat{\gamma}, \gamma$ take the same values as above, we have

F	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
$\pi(F,\rho)$	0.0016	0.0089	0.0164	0.0239	0.0314	0.0390	0.0467	0.0544	0.0622	0.070
F	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.018	0.019	0.020
$\pi(F,\rho)$	0.0779	0.0859	0.0940	0.1021	0.1102	0.1184	0.1267	0.1351	0.1435	0.1520

We can see that $\pi(F,\rho)$ is strictly increasing and (slightly) convex in F.

Lastly, we examine the case in which the value of the AIP is stochastic at $t = 0$. Let $\theta_P = 0.8, \theta_N = 0.1, \theta_U = 0.1, \hat{\gamma} = 0.7, \gamma = 0.4$ and $\rho = 0.36$. For each case, we assume $\tilde{F}$ follows uniform distribution.

1. $F \in [0,1)$. It can shown that $\Pi_{CON} = 0.9848$ and $\Pi_{ABCP} = 0.5776 - S$. It is clear that $\Pi_{CON} > \Pi_{ABCP}$. Thus, convertible debt is preferred.
2. $F \in [0,4]$. Hence, $\bar{F} = 2$. It can be shown that $\Pi_{CON} = 2.0394$ and $\Pi_{ABCP} = 2.1371 - S$. Thus, if $S > 0.0977$, convertible debt is preferred; if $S < 0.0977$, ABCP is preferred.
3. $F \in [0,1.8]$. Hence, $\bar{F} = 0.9$. It can be shown that $\Pi_{CON} = 0.9871$ and $\Pi_{ABCP} = 1.0251 - S$. Thus, if $S > 0.038$, convertible debt is preferred; if $S < 0.038$, ABCP is preferred. □

Proof of Proposition 7: Denote the firm's likelihood of issuing equity as LE. Note that

$$LE = \Pr(\rho \leq \rho^*).$$

It is clear that LE is increasing in ρ since $\Pr(\rho \leq \rho^*)$ is increasing in ρ. □

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Figure 1: **SEQUENCE OF EVENTS**

$t = 0$

- The manager has a project that needs $1 financing from the investor.
- The manager chooses the optimal security design to raise the financing.

$t = 0.5$

- The value of the AIP (assets in place) is realized.
- If the optimal security design is convertible debt, then the investor converts certain amount of debt into equity depending on the realized value of the AIP.

$t = 1$

- The common signal regarding the project quality is realized.
- The manager's and the investor's prior beliefs regarding the signal precision are realized. Potential disagreement may arise.
- The manager decides whether to invest in the project.
- The investor may block the manager in the investment decision according to the control rights allocation specified in the contract of the optimal security design. If the project is not invested, then the $1 financing is returned to the investor.

$t = 2$

- If the project was implemented, its cash flow is realized.
- The total cash flow of the firm is divided between the manager and the investor according to the cash-flow sharing rule specified in the contract of the optimal security design.