

TO BE A PUBLIC COMPANY OR NOT TO BE ?

by

Arnoud W. A. Boot ^{*}, Radhakrishnan Gopalan [†], and Anjan V. Thakor [‡]

February, 2004

^{*}University of Amsterdam and CEPR, Faculty of Economics and Econometrics, Roetersstraat 11, 1018WB Amsterdam, The Netherlands; E-mail: a.w.a.boot@uva.nl

[†]PhD Student, University of Michigan Business School, 701, Tappan Street, Ann Arbor, MI 48109; E-mail: gopalanr@umich.edu

[‡]John E. Simon Professor of Finance, Olin School of Business, Washington University in St. Louis, Box 1133, One Brookings Drive, St. Louis, MO; E-mail: thakor@olin.wustl.edu

ABSTRACT

In this paper we analyze a publicly-traded firm's decision to stay public or go private in a setting in which managerial autonomy from shareholder intervention affects the supply of productive inputs by management. With public ownership, the shareholder base of the firm is subject to stochastic shocks as market liquidity facilitates active trading. With heterogeneity among shareholders, this exposes management to uncertainty regarding the extent of shareholder intervention in future management decisions and consequently curtails managerial incentives to supply privately-costly inputs. By contrast, private ownership provides a stable shareholder base and improves these input-provision incentives. Thus, capital market liquidity has a surprising "dark side" that discourages public ownership. This means our model takes seriously a key difference between private and public equity markets in that, unlike the private market, the degree of investor participation and the heterogeneity among shareholders are both stochastic in the public market. This allows us to draw predictions about the effects of investor participation and stock price volatility on the public firm's incentives to go private. We thus provide a link between *investor participation* and *firm participation* in public markets.

TO BE A PUBLIC COMPANY OR NOT TO BE ?

“The public be damned”

Wott Vanderbilt, Reply to Reporter, 1883

1 INTRODUCTION

When should a publicly-traded firm decide to go private? This question, which we address in this paper, is of central importance in the theory of the firm, and one that has been brought into sharper focus by the events of the past few years. During the 1990s, scores of companies went public, with initial public offerings (IPOs) sold at breathtaking prices. However, since the precipitous decline of the stock market, the “going-public” wave appears to have been replaced by the “going-private” wave. In 2002, 66 publicly-traded U.S. firms delisted, compared to 35 in 1999. Many have conjectured that the decline in stock prices after 2000 has discouraged investor participation in public capital markets, and in this way fuelled a “going private” wave by firms. There is no formal theory, however, that provides any justification for this conjecture.

We have also recently witnessed changes in the corporate governance of publicly-traded firms, most notably the passage of the Sarbanes-Oxley Act. This Act, passed in the wake of accounting scandals and other corporate abuses at highly-visible publicly-traded firms, is intended in part to restore investor confidence in the public stock market and ensure continued investor participation. Investor participation in public capital markets thus occupies center stage both from a market perspective – the impact of declining prices on investor participation– and from a regulatory, corporate governance perspective.

The interesting unanswered question is: how will it affect firm participation? We explore the potential interaction between investor participation and firm participation in the context of a publicly-traded firm's choice of whether to stay public or go private.

We model a publicly-traded firm that has assets in place that are known to generate a cash flow in future. In addition, the firm will have future access to a restructuring opportunity that may be either good or bad. A good restructuring opportunity increases firm value and a bad opportunity diminishes firm value. When the restructuring opportunity arrives, the manager, who owns some shares, as well as the other shareholders receive a common signal about whether the opportunity is good or bad. Priors about the precision of this signal that the shareholders and the manager are endowed with are drawn randomly and could be different for the shareholders and the manager. Consequently, it is possible that the manager's posterior beliefs about the value of the restructuring opportunity will diverge from the shareholders' posterior belief. This can lead to disagreement about the optimal course of action.¹ We show that the optimal corporate governance regime will generally involve the manager being given some decision making leeway or autonomy to overcome such disagreement and pursue the decision he thinks is best when investors disagree. We examine the endogenous determination of this degree of managerial autonomy and how it varies across public and private ownership. With private ownership, the ownership base is relatively stable because the relatively high liquidity costs associated with private ownership discourage investors from frequent trading in the shares. This leads to relative stability in the autonomy given to the manager by investors. In the public capital market, the autonomy given to

¹There is nothing radical about the assumption of heterogenous priors. Morris (1995) argues that such a specification is not inconsistent with standard theories of Bayesian decision making and many well-known models, such as the Arrow-Debreu-Mackenzie model as well as Diamond (1967), Linter (1965) and Ross (1976) have permitted heterogeneous priors.

the manager is stochastic because the composition of shareholders is not fixed; it changes as investors trade in and out of the firm's shares. Some of the trading is motivated by a stochastic liquidity shock that investors experience. When investors sell their shares in the public market, we assume that they incur negligible transactions costs, in contrast with the significant costs incurred by investors for trading in the private equity market

In determining managerial autonomy through the stringency of corporate governance, shareholders face the following tradeoff. On the one hand, shareholders want to make governance as stringent as possible because this minimizes the likelihood of the manager making a restructuring decision they don't like. On the other hand, greater stringency leads the manager to supply lower search effort in uncovering a good restructuring opportunity. We show that this tradeoff that determines the optimal degree of managerial autonomy depends on the propensity of shareholders to agree with the manager. A key assumption in our analysis is that of investor heterogeneity. Different investors may have different degrees of propensity to agree with the manager; which means there is potential cross-sectional heterogeneity in the degree of managerial autonomy investors view as optimal for the firm and also in the valuations investors put on the firm. For publicly-traded firms, corporate governance is determined largely by securities markets regulators and hence managerial autonomy is set to be optimal for the "average" firm. By contrast, in a private firm, corporate governance is determined by a private investor, so managerial autonomy can be chosen to be precisely optimal for the firm in question. In deciding whether to take a publicly-held firm private, the manager trades off the higher liquidity of public ownership against the more precisely determined corporate governance stringency of private ownership.

This approach leads to results that square well with stylized facts and others that are new testable predictions. Our main results are as follows. First, we show that the likelihood of a publicly-traded firm going private is decreasing in the level of its stock price, and increasing in the volatility of its stock price and in the liquidity-based cost of trading in the private equity market relative to the cost of trading in the public equity market. Second, an increase in investor participation in the public equity market leads to an increase in the attractiveness of public ownership. The reason is that greater investor participation has two effects. One is that it tends to increase the firm's stock price because we show that the marginal investor holding the firm's stock has a higher valuation when there are more investors. The other is that the (cross-sectional) variance of investor valuations is shown to be smaller when there are more investors. Both effects work in concert to make public ownership more attractive as more investors participate. Third, as the variance of investor valuations increases, not only does public ownership become less attractive for the manager, but the probability of finding an investor to take the firm private also increases. Hence, the probability of a successful going-private transaction is also increasing in the cross-sectional variance of investor valuations. Forth, we show that public firms will be taken private only at a substantial premia above the pre-transaction stock prices. Finally, there is a "natural" counterbalance to the forces that propel firms towards private ownership. As more firms go private, the attractiveness of private ownership for a public firm declines.

Our work is related to a growing body of research on the determinants of public versus private ownership.² It is generally acknowledged that public ownership comes with liquidity benefits and

²See for example Zingales (1995).

that private ownership offers better control (Bhide (1993) and Coffee (1991)).³ This view has been challenged by Bolton and von Thadden (1998) and Maug (1998), who suggest that even public firms could exercise effective control over management. Also, as Pagano (1993) has recognized, corporate governance arrangements with private ownership can be tailored to the needs of a particular firm. Public ownership, on the other hand, offers a more generic governance arrangement. Boot, Gopalan and Thakor (2003) also use this insight to explore an entrepreneur's choice between private and public ownership. In a slightly different vein, Pagano and Roell (1998) argue that excessive monitoring with private ownership may encourage firms to go public, whereas Burkhardt, Gromb and Panunzi (1997) suggest that a tradeoff exists between the ex post efficiency of shareholder control and its adverse ex ante effects on managerial incentives. Some of these features are also part of our analysis, although the tradeoffs are quite different. Moreover, the existing literature has not focused, on the link between investor participation in the public equity market and a publicly-traded firm's choice of whether to go private, not has it explained why a sufficiently low stock price would induce a public firm to go private. These issues are the primary focus of our analysis.⁴ This focus and our assumption of universal risk neutrality distinguish our paper naturally from the contributions of Admati, Pfleiderer and Zechner (1994), Pagano (1993), and Shah and Thakor (1988) who focus on the diversification and risk-sharing benefits of public markets. To summarize, the key distinction between our paper and the existing literature lies in the numerous new testable predictions that our analysis generates.

³This improved investor control may permit investors to limit managerial expropriation of shareholder wealth or other forms of abuses such as those discussed by Jensen (1989). The point that private ownership offers better control has also been made by Black and Gilson (1998).

⁴The disclosure literature has examined the link between investor participation and disclosure (see Bhattacharya and Nicodono (2002), for example). Maksimovic and Pichler (2001) argue that public ownership is accompanied by less flexible disclosure requirements than private ownership. Chenannur and Fulghieri (1999) argue that the public capital markets involve duplicated monitoring costs.

The rest of the paper is organized as follows. Section 2 contains a description of the model. Section 3 has the analysis, Section 4 discusses the empirical predictions and Section 5 concludes. All proofs are in the Appendix.

2 MODEL DESCRIPTION

In this section we describe the key elements of our model: the agents and the project restructuring possibilities, the liquidity cost faced by investors and the managerial autonomy parameter. We conclude this section with the sequence of events.

2.1 Agents and Project Possibilities

We consider an economy with universal risk neutrality and a risk free interest rate of 0. Consider a firm in this economy with several investors. The firm is managed by a manager who owns α percent of the equity of the firm. The firm has publicly listed shares and the balance $1 - \alpha$ percent of the shareholding is held by public investors. There are four dates 0, 1, 2 and 3, defining three time periods. At $t = 0$ the firm has existing assets, which yield a sure cash flow of $S > 0$ at $t = 3$. The firm also has a restructuring opportunity (“project” from now) at $t = 0$. If this project is taken, it will alter the cash flows to be realized from the existing assets. The restructuring does not require any monetary investment but requires the manager to invest privately-costly effort at $t = 1$. The actual restructuring is carried out at $t = 2$ with the altered cash flows being realized at $t = 3$. At every stage, the manager makes decisions to maximize his terminal wealth. Since he owns a fraction of the firm, the long-term interests of the manager and outside financiers are aligned and there is no agency problem related to a

divergence of objectives between the manager and public shareholders.⁵

There is uncertainty about the quality of the project proposed by the manager. The project can be one of two types: Good (G) or Bad (B). A type- G project increases the cash flows from the existing assets from S to X_G , while a type- B project reduces the cash flows from S to 0. The manager can invest effort at $t = 1$ to increase the probability of finding a G project. That is, if the manager invests effort e at a private cost of $\frac{\beta e^2}{2}$, the probability of finding a G project is e , where $\beta > 0$ is a constant. Conditional on e , this probability is common knowledge and represents everybody's prior beliefs about project quality.

The restructuring decision at $t = 2$ depends on the quality of the project proposed by the manager. Both the manager and the investors want to implement a type- G project and want to avoid a type- B project. The manager and investors decide on the optimal choice based on a public signal, z , that they both observe at $t = 2$. The signal value can be G or B , indicating the project quality. Although the manager and investors observe the *same* signal, they have potentially different priors about the informativeness of the signal. The prior beliefs about the informativeness of the signal, q , are drawn randomly from the set $\{I, U\}$, where $q = I$ represents an "informative" signal and $q = U$ represents an "uninformative" signal. The probability of the manager drawing I is $\theta \in (0, 1)$ and of drawing U is $1 - \theta$. When the prior belief is that the signal is informative, the agent believes $Pr(\text{Project } G | s = G, q = I) = 1$. And when the prior belief is that the signal is uninformative, the agent believes $Pr(\text{Project } G | s = G, q = U) = e$, i.e. an uninformative signal fails to change the agent's

⁵We shy away from agency problems not because we believe they are unimportant but because their effects are well understood.

prior beliefs about the quality of the project.

If the public signal is $z = B$, then it is clear that a prior belief that the signal is I would lead to a decision to reject the project. But if the prior belief is that the signal is U , the NPV of the project is estimated by the agents as $e(X_G - S) + (1 - e)(-S)$. If $e \geq \frac{S}{X_G}$, then the agent will wish to implement the project even if he believes that the signal is uninformative. In the following analysis, we rule out this possibility and assume that β is sufficiently large such that $e < S/X_G$ in all cases considered. Hence, whenever an agent believes that the signal z is uninformative, the agent will not wish to implement the project.

If the public signal is $z = G$, and the prior belief is that the signal is I , then the project NPV will be $X_G - S > 0$, and the decision will be to accept the project. If the prior is U , then the project NPV will, be assessed as $e(X_G - S) - [1 - e]S < 0$, and the decision will be to reject the project.

We assume that the manager receives a private signal that is a random draw from $\{I, U\}$ and this signal sets his prior belief about the precision of the public signal z . Similarly, investors as a group also randomly draw a prior belief from $\{I, U\}$ about the precision of the public signal. We allow the prior beliefs about signal precision that the manager and investors draw to be correlated. That is, we assume $Pr(q_i = j|q_m = j) = \rho$, $Pr(q_i = j|q_m = k) = 1 - \rho$, where $j \neq k$, $j, k \in \{I, U\}$, q_i represents the signal precision for the investors and q_m the signal precision for the manager.

It is clear that if $z = B$ is the commonly-observed public signal, the manager and investors will always agree that the project should be rejected, regardless of their prior beliefs about the precision

of the signal. If $z = G$ and investors and the manager have prior beliefs $q = U$, they will once again agree that the project should be rejected. If $z = G$ and investors' prior belief about signal precision is $q = I$ whereas the manager's prior belief is $q = U$, then the manager will wish not to implement even though investors would like to. We assume that no one can force the manager to implement when he does not want to, so disagreement is irrelevant in this case.⁶ However, when $z = G$, the manager's prior belief about signal precision is $q = I$ and the investors' prior belief is $q = U$, the manager will wish to implement but investors won't. This disagreement is the focus of our analysis. It follows that, conditional on the manager and investors observing G and the manager believing that the precision of the public signal is $q = I$, ρ is the probability that the investors agree with the manager that investment in the project should occur. Note that the potential disagreement between the manager and investors is not due to incomplete information aggregation or asymmetric information since everybody starts out with the same priors about project quality and their belief revisions are based on observing the *same* signal.⁷

The greater is the value of ρ , the greater is the likelihood of agreement between the manager and the investors; $\rho = 1$ indicates perfect agreement while $\rho = 0$ indicates perfect disagreement. The agreement parameter ρ can be thought of as being affected by the attributes of the project (or more generally the nature of the firm's business), management's experience and track record in managing projects of that

⁶This is a fairly natural restriction. One way to think about it formally is to assume that the probability with which a project arrives at $t = 2$ is less than 1 and the manager is the first one to see whether the project has arrived, observes z and q , and then decides whether to bring the project forward for investor to evaluate. Only then can investors observe z and their own q . This way the manager can always say there was no project when he does not wish to invest. Adopting this structure would complicate the analysis but would not change our results.

⁷Thus, our focus here is on "genuine" disagreement, which can arise due to a variety of reasons other than the one we focus on, such as non-uniform priors over project value (Allen and Gale (1999) and Morris (1995)), overconfidence on the part of either management or investors (Daniel, Hirshleifer and Subramanyam (1998)), or other psychological reasons like differences in intuition (Clarke and Mackaness (2001)).

type and also the general level of investor confidence in the corporate sector. If the project is one that the manager has dealt with before successfully and investors are familiar with as well, ρ will tend to be high. For unfamiliar projects and/or those the manager may not have successfully handled in the past, ρ can be expected to be low. Since project familiarity and prior experience with management can be expected to vary among investors in the public capital market, there will be heterogeneity among investors in the extent of their agreement with the manager. Since we have a one-firm economy, our firm can be thought of as the representative firm and the level of ρ as the level of agreement of the public capital markets with the representative firm's manager. The level of agreement of the public investors with the representative firm's manager will vary through time depending on the level of investor confidence and the prior performance of the firm. During periods when the prior performance of the firm is good and investor confidence in the manager is high, ρ will tend to be high, in periods preceded by poor firm performance and characterized by low investor confidence in the manager, ρ will tend to be low. Thus, both idiosyncratic factors (firm performance) and systematic factors (investor sentiment) impact ρ .

We explicitly model heterogeneity of ρ across investors by assuming that the public market for a particular firm at any point in time is comprised of N investors and that the ρ for each investor is an independent draw from a continuous probability distribution $G(\rho)$, and an associated density function $g(\rho)$ with support $[\rho_l, \rho_h]$. As we will show later, the valuation of the firm by the investors is increasing in ρ . Hence, at any point in time the shares of the firm will be held by the investors with the highest ρ , since these are the investors with the highest firm valuation. This fact is important and provides interesting dynamics of the "market ρ " with investor participation. We now discuss two key

features of the model: the liquidity costs faced by investors and the corporate governance (or managerial autonomy) parameter η .

2.2 Liquidity cost

In period 2 (between $t = 1$ and 2), investors (including the manager) may suffer a stochastic liquidity shock that compels them to sell their assets to raise cash. They will wish to sell these shares to the investors with the highest valuation. Since sale with public ownership occurs in a well-defined market place finding these investors is greatly facilitated. On the other hand with private ownership finding these investors would involve search costs. For simplicity we normalize the search costs to 0 with public ownership. With private ownership, we argue that the investor's liquidity cost is proportional to firm valuation, since a selling shareholder would engage in more intensive search for more valuable firms and incur more search costs.

2.3 Managerial Autonomy Parameter

Whenever there is disagreement between shareholders and the manager about project quality, there has to be a rule to resolve the disagreement. We model this using the concept of "autonomy" for the manager. Managerial autonomy (parameterized by η) is the probability that the manager will be able to invest in the project in the face of disagreement with the shareholders, and $1 - \eta$ is the probability with which shareholders can successfully intervene to stop the manager from investing in the project when they believe it is not worthwhile. One could also interpret η , which is endogenously determined in our analysis, as the degree of control given to the manager. *Ceteris paribus* the manager prefers more

autonomy to less. This is *not* because he has any innate preference for control, but because greater autonomy gives him a greater ability to do what he believes will maximize firm value. In real world terms, one can think of the manager increasing his autonomy by limiting the information disclosure to shareholders, having more seats on the Board of Directors that can be occupied by directors friendly to the manager, delivering consistently good performance that induces shareholders to give him more elbow room, and so on. In the public markets, η is usually imposed on an individual firm and is designed (endogenously) by capital market regulators for the “average firm” in the economy. By contrast, η with private ownership is endogenously chosen for *each* firm to maximize its value.

2.4 Summary of Sequence of Events

The time line is as follows. At $t = 0$, the manager, who is running a publicly-traded firm, decides whether to take it private or keep it public. If the manager chooses private ownership, then he identifies a private investor at $t = 1$ to buy out all the outstanding public shareholding at $t = 1$ and take the firm private; If this private investor is unsuccessful in buying out the shares, the firm remains public. If the manager chooses to remain public at $t = 0$, then the firm remains public until $t = 3$. After implementation of the chosen ownership structure, the manager invests in effort at $t = 1$ to find a good restructuring project. Between $t = 1$ and $t = 2$, investors and the manager receive their common public signal about project quality and also draw their private priors about the precision of the public signal. At $t = 2$, the restructuring decision is made. As indicated earlier, the only state in which there is decision-relevant disagreement between investors and the manager is when the public signal is G , investors draw a prior belief about its precision that it is $q = U$ and the manager draws a prior

belief about precision that it is $q = I$. In this case, the manager wants to implement the project but investors don't; with probability η , the manager is able to invest in the project despite investor objections. Subsequently, between dates $t = 2$ and $t = 3$, investors could suffer a liquidity shock. If the firm is publicly traded, investors sell their shares in the market at the market-clearing price. If the firm is private, investors will conduct a search to identify the investor who has the maximum valuation for the shares and sell to that investor. Thus with private ownership, there are transaction/search costs involved in selling the shares. Terminal cash flows are realized at $t = 3$. This sequence of events is shown in *Figure 1*.

————— FIGURE 1 GOES HERE —————

3 ANALYSIS

There are two decisions to be made by the manager. One is the choice of whether to stay public or take the firm ownership, is made at $t = 0$. And the other decision is the choice of effort level, which is made at $t = 1$ subsequent to the implementation of the chosen ownership structure. Since these decisions are made sequentially, we analyze them using backward induction. We first analyze the manager's choice of effort for a given ownership mode choice and then go back and analyze the choice of ownership mode by comparing the relevant payoffs. The manager chooses the ownership mode and effort level so as to maximize his expected wealth at $t = 3$. We now analyze the manager's choice of effort and his payoffs under alternate ownership modes.

3.1 Public Ownership

If the manager chooses public ownership at $t = 0$, then there is uncertainty about the identity of shareholders at $t = 2$ and also about the level of agreement between the manager and the shareholders at that time. Since the restructuring decision is taken at $t = 2$, it is the agreement level at $t = 2$ that matters. We now analyze the manager's decision by estimating his effort and payoff at $t = 1$. (As there is no uncertainty in the payoff for the manager with public ownership, his payoffs at $t = 1$ and $t = 0$ are the same.) If the manager invests e in search effort, his payoff at $t = 1$ with public ownership is given by:

$$\begin{aligned}
 V_{pub}^M &= \alpha(1 - \delta)Pr(z = G)Pr(q_m = I)Pr(q_i = I|q_m = I)[X_G - S] \\
 &\quad + \alpha(1 - \delta)[Pr(z = G)Pr(q_m = I)Pr(q_i = U|q_m = I)\eta[X_G - S] + S] - \frac{\beta e^2}{2} \quad (1)
 \end{aligned}$$

Note that the first term is the increase in payoffs of the firm when the investors and the manager see a public signal $z = G$, and both draw prior beliefs that the signal is informative and hence agree on implementing the restructuring. The second term is the increase in cash flows of the firm when a public signal $z = G$ is observed, the manager's prior belief is that the signal is informative and the investors' prior belief is that it is uninformative. In this case the manager is able to undertake the restructuring with a probability η . The next term is the payoff from the existing assets. The penultimate term is the private cost of effort for the manager, and the last term is the liquidity cost in the public market. Note that in all states other than those reflected in the first two terms, the restructuring is not carried out and the assets result in a cash flow of S . Here we have assumed that the η is exogenously imposed. Once we have evaluated the manager's effort choice function, we will go back to endogenize the public

market η to maximize the share-price at $t = 1$. Thus,

$$\begin{aligned}
V_{pub}^M &= \alpha(1 - \delta)e\theta\bar{\rho}[X_G - S] + \alpha(1 - \delta)[e\theta(1 - \bar{\rho})\eta[X_G - S] + S] - \frac{\beta e^2}{2} \\
&= \alpha(1 - \delta)\{e\theta\bar{\rho}[X_G - S] + e\theta(1 - \bar{\rho})\eta[X_G - S] + S\} - \frac{\beta e^2}{2} \\
&= \alpha(1 - \delta)\{e\theta[\bar{\rho} + [1 - \bar{\rho}]\eta]A - D\} - \frac{\beta e^2}{2}
\end{aligned} \tag{2}$$

where $A = X_G - S$ and $D = -S$.

Here A and D represent the NPV of the project in the agreement and the disagreement states as seen by the investors. $\bar{\rho}$ represents the expected level of agreement between the manager and investors at $t = 2$. The manager chooses his effort at $t = 1$ so as to maximize V_{pub}^M . We assume that the manager's shareholding is sufficient to satisfy the individual rationality (IR) constraint of the manager. We can now show the following.

Lemma 1

The manager's uniquely optimal choice of effort, e_{pub}^* is strictly increasing in $\bar{\rho}$, the expected level of agreement at $t = 2$, α , the managerial ownership fraction, and η , the autonomy parameter, and is strictly decreasing in β , the effort disutility parameter .

Lemma 1 gives a number of intuitive results and also highlights the cost of disagreement. Since the manager knows that the probability that a G -type project discovered due to his effort will actually be implemented is increasing in $\bar{\rho}$, he also expends greater effort when $\bar{\rho}$ is higher. Similar reasoning holds for the impact of the autonomy parameter η . Effort is also increasing in managerial ownership α

because a higher α corresponds to a higher marginal return to effort. The effort level is decreasing in the effort disutility parameter β since this signifies a higher marginal cost of effort.

Given the optimal managerial effort function, the total value of the firm as perceived by the manager at $t = 1$ is given by:

$$\begin{aligned} V_{pub}^M &= \alpha \{ e_{pub}^* \theta [\bar{\rho} + (1 - \bar{\rho}) \eta] A - D \} - \frac{\beta e_{pub}^{*2}}{2} \\ &= \frac{\beta e_{pub}^{*2}}{2} - \alpha D \end{aligned} \quad (3)$$

The value as perceived by the manager is increasing in managerial effort and in the level of managerial shareholding (note $D < 0$). The total value of the firm as perceived by the investors at $t = 1$ (the stock price at $t = 1$) is given by:

$$\begin{aligned} V_{pub}^I &= (1 - \alpha) Pr(s = G) Pr(q_m = I) Pr(q_i = I | q_m = I) [X_G - S] \\ &\quad + (1 - \alpha) [Pr(s = G) Pr(q_m = I) Pr(q_i = U | q_m = I) \eta [-S] + S] - \lambda L_{pub} \\ &= (1 - \alpha) \{ e_{pub}^* \theta [\bar{\rho} [X_G - S] + (1 - \bar{\rho}) \eta [-S]] + S \} \\ &= (1 - \alpha) \{ e_{pub}^* \theta [\bar{\rho} A + (1 - \bar{\rho}) \eta D] - D \} \end{aligned} \quad (4)$$

The level of agreement of the investor at $t = 1$ does not impact the stock price since it is only the level of agreement of the investor at $t = 2$ that matters for firm valuation. V_{pub}^I is increasing in $\bar{\rho}$. Thus, a higher level of agreement connotes a higher stock price. At any point in time the investors most likely to hold the shares in the public market are the ones who have the maximum valuation for the shares. Since the share price is increasing in $\bar{\rho}$, these are the investors with the highest level of agreement with the manager. When ρ varies cross-sectionally and the ρ for any investor is a random

draw from a probability distribution, the highest value of ρ among the N investors is the N^{th} order statistic of ρ , say ρ_N . i.e. $\rho_N = \max_{1 \leq i \leq N} \rho_i$. From now on, whenever we talk about the level of agreement between the manager and shareholders in the public market we mean this maximal level of agreement. The expected value of the agreement parameter represents the expected value of the N^{th} order statistic, i.e. $E(\rho_N) \equiv \bar{\rho}$. We now discuss how the managerial autonomy in the public market is determined.

In the public market, η is determined to maximize V_{pub}^I . A change in η has two opposite effects on V_{pub}^I . According to Lemma 1, managerial effort increases with η . From (4), we know that V_{pub}^I is increasing in managerial effort, so an increase in η leads to an increase in the stock price. However, an increase in η also results in a higher probability of the manager implementing the restructuring that the public shareholders disagree with; this decreases the stock price. To ensure that both effects play an important role and that the stock price is concave in η , we make Assumption 1.

Assumption 1

$$X_G > 2S.$$

Given Assumption 1, we can now establish the following result:

Lemma 2

The optimal managerial autonomy parameter, η_{pub}^* , is given by the following expression:

$$\eta_{pub}^* = \frac{\bar{\rho}(A + D)}{-2(1 - \bar{\rho})D}$$

η_{pub}^* is increasing in $\bar{\rho}$, the level of agreement of the shareholders and the manager at $t = 1$, and X_G , the incremental cash flow from the type- G project, and decreasing in S , the cash flow from the existing assets.

Lemma 2 characterizes the properties of the public market governance regime. First, managerial autonomy is increasing in $\bar{\rho}$. It is intuitive that greater is the propensity of investors to agree with the manager the greater is the autonomy given to the manager by the investors. An increase in $X_G - S$ means an increase in the marginal return to managerial effort, which implies a higher equilibrium level of effort. To get the higher level of effort, investors offer a higher η . Thus managerial autonomy is increasing in $X_G - S$.

3.2 Public Market Governance Regimes and Investor Participation

As we argued earlier at any point in time the shares of the firm are likely to be held by investors with the highest level of agreement with the manager. When ρ varies cross-sectionally, the highest value of ρ among the N investors is ρ_N , the N^{th} order statistic of ρ . Since ρ in the public market is distributed according to the distribution function $G(\rho)$ and a density function $g(\rho)$, with support, $[\rho_l, \rho_h]$, we know from standard results that $\bar{\rho}$, the expected value of the N^{th} order statistic, is given as:

$$\bar{\rho} = \frac{N!}{(N-1)!} \int_{\rho_l}^{\rho_h} G(\rho_N)^{N-1} g(\rho_N) \rho_N d(\rho_N) \quad (5)$$

Lemma 3

$\bar{\rho}$, the expected value of ρ_N is increasing in N and the variance of ρ_N , σ_N^2 , is decreasing in N .

The intuition behind this result is that greater the number of investors in the market, greater will be the propensity of the “maximal” investor to agree with the manager, where the maximal investor is the investor most likely to agree with the manager. Thus $\bar{\rho}$ is increasing in N . Similarly the greater the number of investors in the market small will be the *expected* variation in the level of agreement of the maximal investor.

Thus far we have maintained that the shareholders of the firm will be the investors with the maximum level of agreement with the manager. All our results will go through if the marginal investor in the firm is the investor with the k^{th} largest level of agreement with the manager, for some $k < N$.

From Lemmas 2 and 3 we have the following proposition.

Proposition 1

Greater investor participation in the public capital market leads to a higher mean of the agreement parameter, $\bar{\rho}$, and a higher managerial autonomy parameter, η .

This proposition states that greater investor participation in the equity markets results in corporate governance that appears to be more lax. This is an intuitive result. Greater investor participation leads, on average, to higher agreement between the manager and shareholders (Lemma 3). This leads to greater autonomy being given to the manager. If firms with more analysts following them (i.e. the better-known firms) can be viewed as those with greater investor participation, then this result says that the better-known firms will have more lax corporate governance as represented by greater managerial autonomy. The better-known firms may be the large firms or older firms. In either case,

this result provides a prediction that is potentially testable. Some evidence in support of the proposition is provided by Gompers, Ishi and Metrick (2003) who show that larger firms have a higher value of their governance index, indicating greater protection for managers from takeovers, which can be a proxy for managerial autonomy.

Moreover, Lemma 3 is also interesting because it indicates two important effects of higher investor participation. Greater participation not only increases the mean of the agreement parameter, but (in the special case of uniform density) also reduces its variance. So stock price volatility should go down as more investors participate, since there is a one-to-one relationship in our model between the volatility of ρ and the volatility of stock prices.

Managerial autonomy, the level of managerial effort, the managerial payoff and the stock price at $t = 1$ with public ownership are given as follows:

$$\eta_{pub}^* = \frac{\bar{\rho}(A + D)}{-2(1 - \bar{\rho})D} \quad (6)$$

$$e_{pub}^* = \frac{\alpha\theta\bar{\rho}A(A - D)}{-2\beta D} \quad (7)$$

$$\begin{aligned} V_{pub}^I &= (1 - \alpha)\{e_{pub}^*\theta[\bar{\rho}A + (1 - \bar{\rho})\eta_{pub}^*D] - D\} \\ &= (1 - \alpha)\left\{\frac{\alpha\theta^2\bar{\rho}^2A(A - D)^2}{-4\beta D} - D\right\} \end{aligned} \quad (8)$$

$$\begin{aligned} V_{pub}^M &= \frac{\beta e_{pub}^{*2}}{2} - \alpha D \\ &= \frac{\alpha^2\theta^2\bar{\rho}^2A^2(A - D)^2}{4\beta D^2} - \alpha D \end{aligned} \quad (9)$$

Lemma 4

The managerial payoff with public ownership is increasing in the level of investor participation in

the public equity markets.

This lemma states that greater investor participation makes public ownership more attractive for firms. This is intuitive. Greater investor participation increases the probability of finding investors with higher levels of agreement with the manager and hence higher valuations.

3.3 Private Ownership

If the manager chooses to have private ownership at $t = 0$, then a private investor will try to take the firm private at $t = 1$. With private ownership, there is uncertainty about the identity of the private investor at $t = 0$ as the private investor is identified only at $t = 1$. Further with private ownership, the level of managerial autonomy is set to maximize firm valuation from the point of view of the private investor. The other difference between private and public ownership is the search costs that the private investor would have to incur to sell the shares. i.e. if the private investor were to get a liquidity shock between $t = 2$ and $t = 3$, he would attempt to sell the shares. He would search for the investor with the highest valuation of the shares and in the process incur search costs. The search costs can be given as equal to λL_{pri} . Here λ denotes the probability of the investors suffering a liquidity shock, while L_{pri} denotes the search costs incurred by the investors when they try to sell their shares. The search costs will be incurred only when it pays for shareholders to search out an investor with a high ρ . These are the states in which the restructuring is carried out. In the states in which restructuring is not carried out, the value of the firm is S and does not depend on the level of agreement between the manager and the investor. We now provide a formalization of the search costs endogenously.

3.4 A Formalization of search costs

When an investor wishes to sell his shares he searches for the investor with the highest valuation.

This involves individually searching for investors in the private market.

The private investor can sample an investor at random from the possible pool of investors and discover that investor's valuation. Depending on the valuation of the new investor, the private investor can either sell the shares to the new investor or continue searching. Each search costs c for the private investor. Further, discovering the valuation of the new investor also helps the private investor identify the investors who are likely to have higher valuations than the new investor, so the private investor is able to target his next sampling better.⁸ We also assume that if the second investor sampled has a ρ lower than the first investor sampled, the potential seller can come back to the first investor sampled. Thus, sampling creates an option for the private investor who wishes to sell. When the private investor samples an investor with $\rho = \tilde{\rho}$, the valuation of the firm by the new investor is equal to $\tilde{\rho}eX_h$. This is so because we have assumed that the investor suffers a liquidity shock only after the project has been undertaken. As we discussed earlier, search for a new investor is warranted only in states in which the project is undertaken. The expected value of the search option can be given as follows:

$$\begin{aligned} & E(\rho | \rho \geq \tilde{\rho})eX_h - \tilde{\rho}eX_h \\ &= \left[\left(\frac{\rho_h + \tilde{\rho}}{2} \right) - \tilde{\rho} \right] eX_h \end{aligned} \tag{10}$$

Here $\tilde{\rho}$ indicates the level of agreement of the investor who has currently been sampled by the private

⁸We make this assumption specifically to simplify the algebra. The main results that we want, namely that the search costs are increasing in firm valuation, will be obtained without this assumption. But with this assumption, we get a particularly simple characterization of the search costs, which greatly simplifies our algebra.

investor. The option value is decreasing in $\tilde{\rho}$. The private investor will continue sampling as long as this value is less than or equal to δ , the cost of sampling. i.e.

$$[(\frac{\rho_h + \tilde{\rho}}{2}) - \tilde{\rho}]eX_h \leq \delta ,$$

$$\text{which implies } \tilde{\rho} \geq \rho_h - \frac{2\delta}{eX_h}$$

Thus, the private investor will continue searching until he finds an investor with a ρ greater than or equal to the cut-off indicated, ρ_c . The expected cost of this search can be calculated as follows:

$$\begin{aligned} L_{pri} &= \delta Pr(\tilde{\rho}_1 \geq \rho_c) + 2\delta(1 - Pr(\tilde{\rho}_1 \geq \rho_c))Pr(\tilde{\rho}_2 \geq \rho_c) + 3\delta \prod_{i=1}^2 (1 - Pr(\tilde{\rho}_i \geq \rho_c))Pr(\tilde{\rho}_3 \geq \rho_c) + \dots \\ &= \delta Pr(\tilde{\rho}_1 \geq \rho_c)[1 + 2(1 - Pr(\tilde{\rho}_i \geq \rho_c)) + 3(1 - Pr(\tilde{\rho}_i \geq \rho_c))^2 + 4(1 - Pr(\tilde{\rho}_i \geq \rho_c))^3 + \dots] \\ &= \frac{\delta Pr(\tilde{\rho}_1 \geq \rho_c)}{(1 - (1 - Pr(\tilde{\rho}_i \geq \rho_c)))^2} \\ &= \frac{\delta}{Pr(\tilde{\rho}_i \geq \rho_c)} \\ &= \frac{\delta \Delta \rho}{\rho_h - \rho_c} \\ &= eX_h \Delta \rho \end{aligned}$$

where $\Delta \rho = \rho_h - \rho_l$. Thus, we see the expected search cost is a linear function of the effort level of the manager.

We shall now analyze the case of private ownership through backward induction. That is, for a given level of agreement between the manager and investor at $t = 1$, we shall evaluate the manager's payoff and the investor's payoff and the optimal level of managerial autonomy. Then we shall go back to $t = 0$ and evaluate the manager's payoff and analyze his choice between private and public ownership. Given the level of agreement of the private investor with the manager, $\tilde{\rho}$, and the level of managerial

autonomy $\tilde{\eta}$, the optimal managerial effort choice (e_{pri}) is given by:

$$e_{pri}^* = \frac{\alpha\theta}{\beta}[\tilde{\rho} + (1 - \tilde{\rho})\tilde{\eta}]A \quad (11)$$

Given the level of managerial effort, the payoff for the private investor is given by:

$$\begin{aligned} V_{pri}^I &= (1 - \alpha)\{e_{pri}^*\theta[\tilde{\rho}(X_G - S - \lambda L_{pri}e_{pri}) + (1 - \tilde{\rho})\tilde{\eta}(-S - \lambda L_{pri}e_{pri})] + S\} \\ &= (1 - \alpha)\{e_{pri}^*\theta[\tilde{\rho}A + (1 - \tilde{\rho})\tilde{\eta}D] - [\tilde{\rho} + (1 - \tilde{\rho})\tilde{\eta}]\lambda L_{pri}e_{pri} - D\} \end{aligned} \quad (12)$$

where $L_{pri} = X_h\Delta\rho$. Managerial autonomy with private ownership, η_{pri}^* is set so as to maximize the valuation of the private investor. The following lemma characterizes the managerial autonomy with private ownership.

Lemma 5

The optimal managerial autonomy parameter, η_{pri}^* , is given by the following expression:

$$\eta_{pri}^* = \frac{D - 3C_{pri}\tilde{\rho} + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}{3C_{pri}(1 - \tilde{\rho})} \quad (13)$$

where $C_{pri} = \lambda(A - D)\Delta\rho$. η_{pri}^* is increasing in $\tilde{\rho}$, the level of agreement between the private investor and the manager at $t = 1$, and X_G , the incremental cash flow from the type- G project, and decreasing in S , the cash flow from the existing assets and λ , the probability of liquidity shock.

The intuition for much of this lemma is similar to that for Lemma 2. In addition the intuition for the dependence of η_{pri}^* on λ is as follows. As the liquidity shock becomes more likely, i.e. as λ increases, the expected search costs become greater. Since the search costs are increasing in managerial effort, it is optimal to reduce managerial effort. Since, managerial effort is increasing in η (11), it is optimal

to set a lower η_{pri}^* as λ increases. Given the optimal level of managerial autonomy, the payoffs for the investors and the manager with private ownership, V_I^{pri} and V_M^{pri} respectively, are given as follows:

$$V_{pri}^I = (1 - \alpha)\{e_{pri}^*\theta[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pri}^*D - C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta_{pri}^*]e_{pri}^*] - D\} \quad (14)$$

$$\begin{aligned} V_{pri}^M &= \frac{\beta e_{pri}^{*2}}{2} - \alpha D \\ &= \frac{\alpha^2 \theta^2 A^2}{2\beta} \left[\frac{2D^2 + 3C_{pri}\tilde{\rho}(A - D) + 2D[\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}]{9C_i^2} \right] - \alpha D \end{aligned} \quad (15)$$

The payoffs of the manager and the investor have been evaluated at $t = 1$ after the private investor has been identified. But the manager decides on the ownership structure at $t = 0$ before the private investor has been identified. Since the $\tilde{\rho}$ of the private investor is stochastic at $t = 0$, the expected payoff of the manager at $t = 0$ is evaluated using the expectation over $\tilde{\rho}$. Here again the private investor will be the one with maximal valuation for the firm's shares. Thus, the level of agreement of the private investor is nothing but the N^{th} order statistic. The key is that not all investors will be able to take the firm private. As we will show shortly, there exists a cutoff value of $\tilde{\rho} = \hat{\rho}$ at $t = 1$ such that only investors with $\tilde{\rho} \geq \hat{\rho}$ will be able to take the firm private. If there is no investor with $\tilde{\rho} \geq \hat{\rho}$, then the firm will not go private although the manager might wish to do so. Thus, the expectation of the manager's payoff is taken over a truncated distribution of $\tilde{\rho}$ with support $[\hat{\rho}, \rho_h]$. Let the expected value of the agreement parameter of the possible private investor over this truncated distribution be $\bar{\rho}$, and let the variance of the agreement parameter be $\overline{\sigma_{\tilde{\rho}}^2}$. The expected payoff of the manager with private ownership is given by:

$$\begin{aligned} E(V_{pri}^M) &= E\left[\frac{\beta e_{pri}^{*2}}{2} - \alpha D\right] \\ &= \frac{\beta E(e_{pri}^{*2})}{2} - \alpha D \end{aligned}$$

Substituting for e_{pri}^{*2} we have:

$$\begin{aligned}
&= \frac{\beta E(\{\frac{\alpha\theta A}{\beta}[\frac{D+\sqrt{D^2+3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}}]\}^2)}{2} - \alpha D \\
&= \frac{\alpha^2\theta^2 A^2}{2\beta} \left[\frac{2D^2 + 3C_{pri}\bar{\rho}(A-D) + 2DE[\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}]}{9C_i^2} \right] - \alpha D
\end{aligned}$$

The following proposition characterizes the manager's choice between private and public ownership.

Proposition 2

The probability of the manager choosing private ownership is increasing in $\overline{\sigma_\rho^2}$, the variance of the agreement parameter of the private investor, in $\bar{\rho}$, the expected level of agreement of the private investor, and decreasing in $\bar{\rho}$, the expected agreement parameter in the public market, and the expected liquidity cost of private ownership, C_{pri} .

Proposition 2 highlights the factors that induce firms to go private. Firms will choose private ownership when the volatility of $\tilde{\rho}$ of the private investor is high. Given that the private investor is being sampled from the pool of public investors, this directly implies that firms will choose private ownership when the volatility of the agreement parameter in the public markets is high. The volatility of ρ comes from investor heterogeneity and other factors like investor sentiment; the greater is the heterogeneity among shareholders and the more unpredictable is the valuation of the maximal investor, the greater is the probability that a firm will choose private ownership. Since the market price is a linear function of ρ , this corollary shows that a firm will choose private ownership for a sufficiently high volatility in its stock price. The probability of choosing private ownership is also decreasing in $\bar{\rho}$. Given a distribution of ρ , we showed in Lemma 3 that $\bar{\rho}$ is just a function of N . Thus, when the level of

investor participation in the public markets is low, N will be low and so will be $\bar{\rho}$. This is also when the volatility of ρ in the public market would be the highest (Lemma 3). Thus the incentives for the firms to go private would be highest in these circumstances. The attractiveness of private ownership is increasing in $\bar{\rho}$, the expected level of agreement of the private investor with the manager. As, we mentioned $\bar{\rho}$ is endogenously determined as the expected value of the N^{th} order statistic from a truncated distribution. The attractiveness of private ownership is decreasing in the expected liquidity cost of private ownership, C_{pri} .

Although the manager might prefer private ownership at $t = 0$, it is clear that for the firm to actually go private, a shareholder must be able to buy all the shares from the public market at $t = 1$. The private investor will be able to buy these only if his valuation, given by (14), is greater than the public share price. The share price that will prevail at $t = 1$, when the firm has public ownership, is given by (4). Comparing (4) and (14) we see that an investor will be able to buy out the public shareholding iff the following condition holds:

$$\begin{aligned} & (1 - \alpha)\{e_{pri}^*\theta[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pri}^*D - C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta_{pri}^*]e_{pri}^*] - D\} \\ & \geq (1 - \alpha)\{e_{pub}^*\theta[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pub}^*D - D\} \end{aligned}$$

which implies

$$\begin{aligned} & e_{pri}^*[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pri}^*D - C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta_{pri}^*]e_{pri}^*] \\ & \geq e_{pub}^*[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pub}^*D] \end{aligned} \tag{16}$$

This now allows us to prove the following.

Proposition 3

There exists a cutoff value of the agreement parameter, $\tilde{\rho}$ equal to $\hat{\rho}$ such that only investors with $\tilde{\rho} \geq \hat{\rho}$ will be able to take the firm private. The cutoff $\hat{\rho}$ is increasing in $\bar{\rho}$, the expected level of agreement in the public market, and in C_{pri} the expected liquidity penalty with private ownership.

This proposition shows that only an investor who values the firm (and manager) sufficiently highly (in comparison to the market) will be able to undertake a going-private transaction. From the point of view of this private investor, the main tradeoff between public and private ownership is the lower liquidity cost with public ownership versus the higher level of managerial effort due to stability of shareholder base with private ownership. An obvious implication of this is that a sufficiently high premium will have to be paid to take the firm private if one assumes (as we will later) that the price paid is increasing in the private investor's valuation of the firm. The other benefit of private ownership is the ability to precisely tailor the level of managerial autonomy parameter. But in our setting the optimal level of autonomy from the point of view of an individual investor is different for public and private ownerships. This is because, with private ownership, the liquidity cost depends on the level of autonomy. Since an investor takes this into account while setting the level of autonomy with private ownership, for a given investor, the optimal level of autonomy with private ownership is lower than that with public ownership. Consequently, we will not be able to make any general statements comparing the levels of autonomy and consequent managerial effort with public and private ownership.

Corollary 1

The probability that the firm will go private at $t = 1$ is decreasing in N , the level of investor participation in the public capital markets and decreasing in λ the probability of liquidity shock of the private investor.

Investor participation in the public markets decreases the probability of going private transactions through two effects. First, greater investor participation increases \bar{p} , and this, in turn, increases the firm's stock price and reduces the probability of finding a private investor with a higher valuation. Second, greater investor participation also reduces the variance of ρ in the public markets (Lemma 3). This reduction in variance diminishes the probability of finding an investor with a higher agreement parameter than the one corresponding to the stock market price (since $\bar{\bar{p}} > \bar{p}$), and this again reduces the probability of a successful going-private transaction. It is apparent from this that private ownership will be preferred in situations in which investor's agreement with managerial decisions in the "average" firm (\bar{p}) is relatively low, leading to a relatively low managerial autonomy, η_{pub}^* . Thus, we will have public firms going private more often during periods of low stock prices – characterized by low \bar{p} – than during periods of high stock price. This result is consonant with anecdotal evidence about going-private transactions that we mentioned in the Introduction, as well as the empirical evidence that IPOs are largely a bull-market phenomenon (see, for example, Ritter and Welch (2002)).

An increase in λ increases the liquidity penalty of private ownership and consequently makes private ownership less attractive. This also reduces the probability that the firm would go private.

Because the private investor is buying out all outstanding publicly-traded shares, there will not be any free riding by individual shareholders as in Grossman and Hart(1980). We have also not discussed the price at which the private investor acquires these shares, with the only constraint being that the private investor's valuation should exceed the public market price. This constraint ensures that the going-private transaction generates a surplus. We assume that the public shareholders get a fraction γ of the surplus while the private investor gets $1 - \gamma$. Thus, the price at which the private investor can acquire the shares is $P = V_{pub}^I + \max\{0, \gamma[V_{pri}^I - V_{pub}^I]\}$.⁹

3.5 Cost of capital for the going-private transaction

In this section we extend our basic model by assuming that there is a fixed cost C to be incurred by any investor wishing to become a private investor. This cost can be interpreted as consisting of the fixed costs incurred by the private investor in researching the firm, drawing up the private equity contract, and the costs of raising finance. With this setup we explore the incentives for investors to take firms private and the relationship between the number of firms going private and the cost of capital for a private firm.

We first examine the incentives for an investor to incur the fixed costs of taking the firm private by comparing the payoff from becoming a private investor with the payoff from staying a public investor. The payoff from becoming a private investor is the share of surplus that the private investor gets from taking the firm private. We have assumed that the private investor gets $1 - \gamma$ share of the surplus created by taking the firm private. This surplus is created only if the private investor finds a firm to

⁹We can generate this sort of surplus-sharing rule via an explicit Nash bargaining game.

take private and is successful in taking the firm private. Thus, the benefit from becoming a private investor can be expressed as: $B_{pri} = Pr(Firm) * Pr(Private) * Surplus(Private)$. Here $Pr(Firm)$ is the probability that there will be a firm available for the investor to take private. This probability is increasing in the number of firms that want to go private. $Pr(Private)$ is the probability that the going-private transaction will be successful. $Surplus(Private)$ is the surplus obtained by the private investor in a going-private transaction. Substituting for the known quantities, we have the following condition for an investor to incur the cost of attempting to take a firm private: $B_{pri} = Pr(Firm) * Pr(V_{pri}^I \geq V_{pub}^I) * (1 - \gamma)(max(0, V_{pri}^I - V_{pub}^I))$ Thus, an investor will be willing to incur the cost C if $E(B_{pri}) \geq C$. This gives us our next lemma.

Lemma 6

There exists cutoff value of the agreement parameter $\hat{\rho} = \rho_c$ such that only investors with agreement parameters $\rho \geq \rho_c$ will invest to become private investors. The cutoff value is increasing in $\bar{\rho}$ and decreasing in the number of firms wanting to go private.

Lemma 6 indicates that the investor should have a high degree of agreement with the manager to wish to incur the cost of becoming a private investor. The intuition for the Lemma is as follows. From our earlier analysis, we know that V_{pri}^I is increasing in the ρ of the private investor. Hence, the benefit of becoming a private investor is increasing in the ρ of that investor. Further $E(V_{pub}^I)$ is increasing in $\bar{\rho}$. Thus, the probability of a successful going-private transaction and the surplus created from such a transaction are both decreasing in $\bar{\rho}$. Consequently, the incentives for an investor to become a private

investor is decreasing in $\bar{\rho}$. Similar to Corollary 1, this lemma predicts that private equity activity will escalate during bear markets when stock prices are low. Moreover, this lemma states that the average level of agreement of private investors with the manager is decreasing in the number of firms going private. This is because $\hat{\rho}$, the cutoff value of ρ is decreasing in the number of firms going private; So the average level of ρ among the pool of private investors decreases as more firms go private. The following proposition illustrates the relationship between the number of firms going private and the cost of capital for a private firm and the probability of a successful going-private transaction.

Proposition 4

The cost of capital for a private firm is non-decreasing in the number of firms going private. The probability of a successful going private transaction is non-increasing in the number of firms going private.

This proposition highlights an interesting “self-correcting” property of going-private waves. The intuition for this result is as follows. As more firms want to go private, we have more investors investing to become private investors. This reduces the level of agreement of the marginal private investor with any firm. Consequently, the expected cost of capital for a private firm (which is decreasing in the level of agreement of the private investor with the manager) is increasing in the number of firms going private. Further, it also decreases the expected probability of an average firm undertaking a successful going-private transaction. Thus, from a firm’s point of view, the marginal attractiveness of going-private is decreasing in the number of firms going private, which will cause the going-private wave to eventually

cease.

4 Empirical predictions

We discuss the main empirical predictions of our model. For the most part, these are novel predictions that have yet to be confronted with the data.

1. The greater the investor participation for a publicly-traded firm, the lower should be the stringency of corporate governance for the firm. This prediction follows from Proposition 1. If we view larger firms as having greater investor participation, then this prediction is consistent with Gompers, Ishi and Metrick (2003) that managers in larger firms have greater protection from takeovers, assuming that greater protection signifies more managerial autonomy. However, one would need to more directly test this prediction by using other proxies for investor participation, such as the number of analysts following the firm.
2. The probability of a going-private transaction is decreasing in the extent of investor participation. This follows from Proposition 2 and Corollary 1. The testable prediction is that firms that have a larger investor base, are better known and have more analysts following them are less likely to go private.
3. A public firm will go private when its stock price is sufficiently low and/or the volatility of this price is sufficiently high. This follows from Proposition 3. This prediction is consistent with the anecdotal evidence presented in the Introduction, but we are not aware of any large-sample empirical evidence about it.

4. The probability of going-private transaction declines as the liquidity cost of private ownership increases. This follows from Proposition 3. One way to test this prediction would be to compare going-private transactions across markets or countries with private equity markets with varying levels of liquidity.

5. Going-private transactions will occur more often when the corporate governance for publicly-traded firms is relatively stringent. This follows from Corollary 1, which shows that the probability of going-private is decreasing in $\bar{\rho}$, and Lemma 2 which shows that the expected value of η_{pub}^* is increasing in $\bar{\rho}$. This prediction should be relatively straight forward to test either cross-sectionally or intertemporally, particularly given the corporate governance index developed by Gompers, Ishi and Metrick (2003).

6. A going-private transaction will occur only at a price that represents a substantial premium above the firm's pre-transaction stock price. This follows from Proposition 3 and is consistent with the evidence in Larson (2003) that firms that went private in the first two months of 2003 paid premia ranging from 40% to 80%.

7. Ceteris paribus the higher the price paid to take the firm private the less stringent is the corporate governance put in place (the greater is the autonomy given to the manager) *after* the firm is taken private. This follows from Lemma 2 which shows that η^* with private ownership is increasing in $\hat{\rho}$; and we know that the higher is $\hat{\rho}$, the higher is the private investor's valuation of the firm. Moreover, corporate governance will become less stringent after a publicly-held firm goes private. This follows from Lemma 6.

8. The cost of capital of the private firm is non-decreasing in the number of firms going private. As more firms go private at a certain point in time, the less likely it is for another firm to go private at that point in time. This follows from Proposition 4.

5 CONCLUSION

The development of liquid capital markets for publicly-traded firms, achieved through the growing participation of individual investors, is one of the most noteworthy developments in industrialized economies. The intuitive link in the through process is that greater investor participation leads to greater liquidity which then attracts more firms to go public. Yet, even as the liquidity of public markets keeps growing through time, we observe surges and reversals in the going-public process. In particular, publicly-traded firms that experience sufficiently low stock prices often find it attractive to take their firms private.

In this paper we establish a positive link between investor participation in public markets and the incentives for firms to stay public. This works via two channels. Higher investor participation leads to a higher valuation for the marginal investor holding the stock. And secondly the (cross sectional) variance of investor valuations is smaller when there are more investors. Both effects work in concert and raise the valuation of the firm in the public markets. Our analysis also predicts that the price paid for taking a firm private will involve a substantial premium over the firm's pre-transaction stock price.

A central feature of our analysis is that corporate governance is endogenous and that the ownership mode of the firm determines the three key forces – corporate governance, liquidity and the stability of

the firm's shareholder base – that influence the determination of whether the firm stays public or goes private. We believe this approach can be fruitfully used to address other issues in the theory of the firm, such as the boundaries of the firm and the interaction between firm boundaries and its choice of ownership mode.

A APPENDIX

A.1 Proof of Lemma 1

Since V_{pub}^M is concave in e the first order condition (FOC) is necessary and sufficient for the unique maximum. The optimal effort level e_{pub}^* is determined through the FOC as:

$$\alpha\theta[\bar{\rho} + (1 - \bar{\rho})\eta]A - \beta e = 0$$

which yields

$$e_{pub}^* = \frac{\alpha\theta}{\beta}[\bar{\rho} + (1 - \bar{\rho})\eta]A \quad (\text{A-1})$$

It is now clear that $\frac{\partial e_{pub}^*}{\partial \bar{\rho}} > 0$, $\frac{\partial e_{pub}^*}{\partial \alpha} > 0$, $\frac{\partial e_{pub}^*}{\partial \eta} > 0$ and $\frac{\partial e_{pub}^*}{\partial \beta} < 0$. Q.E.D.

A.2 Proof of Lemma 2

The public market governance regime is fixed so as to maximize the $t = 1$ stock price of the firm. The stock price at $t = 1$ is given by (4). Since V_{pub}^I is concave in η , the first-order condition (FOC) is necessary and sufficient for an unique maximum. We can write V_{pub}^I as:

$$\begin{aligned} V_{pub}^I &= (1 - \alpha)\{e_{pub}^*\theta[\bar{\rho}A + (1 - \bar{\rho})\eta D] - D\} \\ &= (1 - \alpha)\left\{\frac{\alpha\theta}{\beta}[\bar{\rho} + (1 - \bar{\rho})\eta]A\theta[\bar{\rho}A + (1 - \bar{\rho})\eta D] - D\right\} \end{aligned}$$

Differentiating with respect to η we get the first-order condition:

$$\bar{\rho}(1 - \bar{\rho})[A + D] + 2\eta D(1 - \bar{\rho})^2 = 0$$

$$\begin{aligned}\eta^* &= \frac{\bar{\rho}(A+D)}{2(1-\bar{\rho})(-D)} \\ \eta^* &= \frac{\bar{\rho}(X_G - 2S)}{2(1-\bar{\rho})S}\end{aligned}\tag{A-2}$$

It is now clear that $\frac{\partial \eta^*}{\partial \bar{\rho}} > 0$, $\frac{\partial \eta^*}{\partial X_G} > 0$ and $\frac{\partial \eta^*}{\partial S} < 0$. Q.E.D.

A.3 Proof of Lemma 3

We first introduce some notation. Let μ_I^N represent the expected value of the I^{th} order statistic from a sample of size N . Thus, $\bar{\rho} = \mu_N^N$. We will use two steps to show that $\bar{\rho}$ is increasing in N . First, we can show that $\mu_N^N \geq \mu_{N-1}^N$. This result has been proved in Galton and Pearson(1902) who show:

$$\mu_N^N - \mu_{N-1}^N = \frac{N!}{(N-2)!} \int_{\rho_l}^{\rho_h} G(\tilde{\rho})(1-G(\tilde{\rho}))^{N-1} d(\tilde{\rho})\tag{A-3}$$

Since the integral on the right-hand side is positive, we have $\mu_N^N \geq \mu_{N-1}^N$. The second step of the proof uses the following identity of order statistics. It can be shown that:

$$(N-1)\mu_N^N + \mu_N^{N-1} = N\mu_{N-1}^{N-1}$$

From our earlier result we have

$$(N-1)\mu_N^N + \mu_N^N \geq (N-1)\mu_N^N + \mu_N^{N-1} = N\mu_{N-1}^{N-1}$$

which means

$$N\mu_N^N \geq N\mu_{N-1}^{N-1}$$

To prove the second part of the Lemma, we first note that asymptotically ρ_N has the following degenerate distribution with 0 variance.

$$\begin{aligned} F(\rho_N) &= 1 & \rho_N &= 1 \\ &= 0 & \rho_N &\neq 1 \end{aligned}$$

Thus asymptotically ρ_N has 0 variance. To show that for finite N the variance of ρ_N is decreasing in N we can use an approximate inverse Taylor series expansion of the expression of the variance of ρ_N .

i.e. the variance of ρ_N is given as $Var(\rho_N) = E(\rho_N^2) - E(\rho_N)^2$ where

$$E(\rho_N) = \int_0^1 F(\rho)^{N-1} f(\rho) \rho d(\rho) \quad (\text{A-4})$$

we know from the probability integral transformation $F(\rho_N) = u_N$ where u_N is the N^{th} order statistic from an uniform distribution. Thus we can express any function of the N^{th} order statistic of any continuous distribution as a function of the N^{th} order statistic of the uniform distribution. Using this and doing a Taylor series expansion of the variance of ρ_N an approximate expression can be given as:

$$Var(\rho_N) \approx \frac{N}{(N+1)^2(N+2)} \{f[F^{-1}(\frac{1}{N+1})]\}^{-2} \quad (\text{A-5})$$

For a detailed derivation of the above expression please refer to Gibbons(1971). As noted in Gibbons(1971) this approximation is quite good for the N^{th} order statistic. The above expression clearly shows the conditions under which $Var(\rho_N)$ is decreasing in N . We note that $F^{-1}(\frac{1}{N+1})$ is decreasing in N . On the other hand nothing much can be said about $\{f[F^{-1}(\frac{1}{N+1})]\}^{-2}$ unless we know the shape of the distribution. The condition required for $Var(\rho_N)$ to decrease in N is that $\{f[F^{-1}(\frac{1}{N+1})]\}^{-2}$ should not increase with N at an order of magnitude greater than the first term. The first term decreases at an

order of magnitude N^3 . Thus the condition required is that $\{f[F^{-1}(\frac{1}{N+1})]\}^{-2}$ should not increase at an order of magnitude greater than N^3 . Thus under this condition we have our result that the variance of ρ_N is decreasing in N . Q.E.D.

A.4 Proof of Proposition 1

This is obvious from Lemma 2 and Lemma 3. Q.E.D.

A.5 Proof of Lemma 4

It is clear V_{pub}^M is increasing in $\bar{\rho}$. From Lemma 3 we know $\bar{\rho}$ is increasing in N . Q.E.D.

A.6 Proof of Lemma 5

The managerial autonomy parameter with private ownership is fixed so as to maximize the valuation of the private investor. The valuation of the private investor at $t = 1$ is given by (14). Since V_{pri}^I is concave in $\tilde{\eta}$, the first-order condition (FOC) is necessary and sufficient for a unique maximum. We can write V_{pri}^I as:

$$\begin{aligned} V_{pri}^I &= (1 - \alpha)\{e_{pri}^*\theta[\tilde{\rho}A + (1 - \tilde{\rho})\eta D - \lambda L_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta]e_{pri}^*] - D\} \\ &= (1 - \alpha)\left\{\frac{\alpha\theta}{\beta}[\tilde{\rho} + (1 - \tilde{\rho})\eta]A\theta[\tilde{\rho}A + (1 - \tilde{\rho})\eta D - \lambda L_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta]\frac{\alpha\theta}{\beta}[\tilde{\rho} + (1 - \tilde{\rho})\eta]A] - D\right\} \end{aligned}$$

Simplifying we have

$$= [\tilde{\rho} + (1 - \tilde{\rho})\eta][\tilde{\rho}A + (1 - \tilde{\rho})\eta D - \lambda L_{pri}\frac{\alpha\theta A}{\beta}[\tilde{\rho} + (1 - \tilde{\rho})\eta]^2]$$

Differentiating with respect to η we get

$$(1 - \tilde{\rho})[\tilde{\rho}A + (1 - \tilde{\rho})\eta(1 - \tilde{\rho})\eta D] + [\tilde{\rho} + (1 - \tilde{\rho})\eta](1 - \tilde{\rho})D - 3\lambda L_{pri}(1 - \tilde{\rho})\eta]^2(1 - \tilde{\rho}) = 0$$

$$[\tilde{\rho}A + (1 - \tilde{\rho})\eta D] + [\tilde{\rho} + (1 - \tilde{\rho})\eta]D - 3C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta]^2 = 0$$

where $C_{pri} = \lambda K \frac{\alpha\theta A}{\beta} X_h$. This yields

$$\tilde{\rho}(A + D) + 2(1 - \tilde{\rho})\eta D - 3C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta]^2 = 0 \quad (\text{A-6})$$

which implies

$$\eta_{pri}^* = \frac{D - 3C_{pri}\tilde{\rho} + \sqrt{(D - 3C_{pri}\tilde{\rho})^2 - 3C_{pri}(3C_{pri}\tilde{\rho}^2 - (A + D)\tilde{\rho})}}{3C_{pri}(1 - \tilde{\rho})}$$

$$\eta_{pri}^* = \frac{D - 3C_{pri}\tilde{\rho} + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}{3C_{pri}(1 - \tilde{\rho})}$$

The intuition for the result $\frac{\partial \eta^*}{\partial \tilde{\rho}} > 0$ can be had by considering A-6. In the absence of the third term it is clear that $\frac{\partial \eta^*}{\partial \tilde{\rho}} > 0$. We can make the third term as small as possible by having a sufficiently small λ .

We assume that λ is sufficiently small (and so is C_{pri}), such that the third term does not change this result. By the same logic we have $\frac{\partial \eta^*}{\partial X_G} > 0$ and $\frac{\partial \eta^*}{\partial S} < 0$. We shall outline the detailed proof in the case of $\frac{\partial \eta^*}{\partial \tilde{\rho}}$. Differentiating the expression for η_{pri}^* with respect to $\tilde{\rho}$ we get:

$$\frac{\partial \eta^*}{\partial \tilde{\rho}} = \frac{(1 - \tilde{\rho})[-3C_{pri} + \frac{3C_{pri}(A - D)}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}] + [D - 3C_{pri}\tilde{\rho} + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}]{3C_{pri}(1 - \tilde{\rho})^2}}$$

To show $\frac{\partial \eta^*}{\partial \tilde{\rho}} \geq 0$ we need to show

$$(1 - \tilde{\rho})[-3C_{pri} + \frac{3C_{pri}(A - D)}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}] + [D - 3C_{pri}\tilde{\rho} + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}] \geq 0$$

or $D - 3C_{pri} + \frac{3C_{pri}(1 - \tilde{\rho})(A - D)}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}} + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)} \geq 0$

or $D - 3C_{pri} + \frac{3C_{pri}(1 + \tilde{\rho})(A - D) + 2D^2}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}} \geq 0$

Separating the terms and squaring we have

$$\frac{(3C_{pri}(1 + \tilde{\rho})(A - D) + 2D^2)^2}{4D^2 + 3C_{pri}\tilde{\rho}(A - D)} \geq (D - 3C_{pri})^2$$

$$\text{or } 9C_{pri}^2(1 + \tilde{\rho})^2(A - D)^2 + 4D^4 + 12D^2C_{pri}(1 + \tilde{\rho})(A - D) \geq 4[D^2 + 3C_{pri}\tilde{\rho}(A - D)][D^2 + 9C_{pri}^2 - 6DC_{pri}]$$

$$\text{or } 3C_{pri}(1 + \tilde{\rho})^2(A - D)^2 + 4D^2(1 + \tilde{\rho})(A - D) \geq 4\{\tilde{\rho}(A - D)[D^2 + 9C_{pri}^2 - 6DC_{pri}] + D^2(3C_{pri} - 2D)\}$$

which upon simplifying yields :

$$4D^2(A + D) \geq 3C_{pri}\{4[\tilde{\rho}(A - D)][3C_{pri} - 2D] + 4D^2 + (1 + \tilde{\rho})^2(A - D)^2\}$$

The left-hand side is positive and the term within braces in the right-hand side is also positive. The right-hand side can be made as small as required by taking a small enough C_{pri} . To prove $\frac{\partial \eta^*}{\partial \lambda} < 0$, it is sufficient to show $\frac{\partial \eta^*}{\partial C_{pri}} < 0$. This is obvious from differentiating the expression for η^* . Q.E.D.

A.7 Proof of Proposition 2

The manager will prefer private ownership whenever (V_{pub}^M) (equation (3)) is lesser than $E(V_{pri}^M)$ (equation (15)). i.e.

$$\begin{aligned} & \frac{\alpha^2 \theta^2 A^2}{2\beta} \left[\frac{2D^2 + 3C_{pri}\bar{\rho}(A - D) + 2DE[\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]}{9C_{pri}^2} \right] - \alpha D \\ & \geq \frac{\alpha^2 \theta^2 \bar{\rho}^2 A^2 (A - D)^2}{4\beta D^2} - \alpha D \\ \text{or } & \frac{2D^2 + 3C_{pri}\bar{\rho}(A - D) + 2DE[\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]}{9C_{pri}^2} \geq \frac{\bar{\rho}^2 (A - D)^2}{2D^2} \end{aligned}$$

The left-hand side of the inequality above is an increasing convex function of $\tilde{\rho}$. Hence it is increasing in $\bar{\rho}$ and in $\sigma_{\tilde{\rho}}^2$. Further, the left-hand side is decreasing in C_{pri} , while the right-hand side is increasing in $\bar{\rho}$. Q.E.D.

A.8 Proof of Proposition 3

The private investor will be able to take the firm private iff the condition in 16 is satisfied. Rewriting the condition we have:

$$e_{pri}^*[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pri}^*D - C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta_{pri}^*]e_{pri}^*] \geq e_{pub}^*[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pub}^*D],$$

which implies

$$e_{pri}^*[\tilde{\rho}A + (1 - \tilde{\rho})\eta_{pri}^*D - C_{pri}[\tilde{\rho} + (1 - \tilde{\rho})\eta_{pri}^*]e_{pri}^*] \geq \frac{\alpha\theta\bar{\rho}^2 A(A - D)^2}{-4\beta D} \quad (\text{A-7})$$

In the absence of liquidity costs, if the level of agreement of the private investor is equal to $\bar{\rho}$, then the above inequality will be satisfied as an equality. But in the presence of liquidity costs of private ownership, the left-hand side will be strictly lesser than the right hand side for $\tilde{\rho} = \bar{\rho}$. Thus, the inequality will be satisfied only for a $\tilde{\rho} = \hat{\rho} > \bar{\rho}$. The cutoff $\hat{\rho}$ is increasing in $\bar{\rho}$ since the right-hand side is increasing in $\bar{\rho}$, and it is increasing in C_{pri} since the right-hand side is decreasing in C_{pri} . Q.E.D.

A.9 Proof of Corollary 1

The firm will go private whenever the inequality in (A-7) is satisfied. We now substitute for the effort choices and simplify the inequality as follows:

$$\begin{aligned} & \frac{\alpha\theta A[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]}{3C_{pri}\beta} [3\tilde{\rho}(A - D) + \frac{D^2 + D\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}{3C_{pri}}] \\ & - \frac{\alpha\theta A[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]^2}{3C_{pri}\beta} \geq \frac{\alpha\theta\bar{\rho}^2 A(A - D)^2}{-4\beta D} \\ \text{which implies} & \frac{[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]}{3C_{pri}} [3\tilde{\rho}(A - D) + \frac{D^2 + D\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}{3C_{pri}}] \\ & - \frac{\alpha\theta A[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]^2}{3C_{pri}\beta} \geq \frac{\bar{\rho}^2(A - D)^2}{-4D} \end{aligned} \quad (\text{A-8})$$

The only term in the above inequality that depends on N is $\bar{\rho}$ which increases with N . Thus the right hand side increases with N while the left hand side is independent of N . Consequently, the probability of a successful going private transaction is decreasing in N . To show the comparative static with respect to λ , it is sufficient to show that the left hand side is decreasing in C_{pri} since C_{pri} is increasing in λ and the other terms are independent of λ . To show this we can split the left hand side into two terms as follows:

$$\frac{D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}} [3\tilde{\rho}(A-D) + \frac{D^2 + D\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}}] - \frac{\alpha\theta A [D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}]^3}{9C_{pri}^2\beta}$$

We can show that the first term is decreasing in C_{pri} while the second is increasing in C_{pri} . Let the derivative of $\frac{D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}}$ with respect to C_{pri} be Dif . It is apparent that Dif is negative.

Using this notation and differentiating the first expression with respect to C_{pri} we have:

$$Dif [3\tilde{\rho}(A-D) + \frac{D^2 + D\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}}] + \frac{D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)}}{3C_{pri}} [DDif]$$

To show the derivative is lesser than 0 it is sufficient to show that the following is greater than 0.

$$\text{i.e. to show} \quad 3\tilde{\rho}(A-D) + \frac{2D(D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A-D)})}{3C_{pri}} \geq 0$$

Separating the terms and squaring we have

$$\text{to show} \quad (9\tilde{\rho}C_{pri}(A-D) + 2D^2)^2 \geq 4D^2(D^2 + 3C_{pri}\tilde{\rho}(A-D))$$

The last inequality is obviously true. Thus we have shown that the first term is decreasing in C_{pri} . To show that the second term is increasing in C_{pri} we can similarly differentiate it with respect to C_{pri} as

follows:

$$\frac{3C_{pri}^2[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]^2\left(\frac{3\tilde{\rho}(A - D)}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}\right) - 2C_{pri}[D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}]^3}{C_{pri}^4} \geq 0$$

$$\text{or to show} \quad 3C_{pri}\left(\frac{3\tilde{\rho}(A - D)}{2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}}\right) - [D + \sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}] \geq 0$$

$$\text{Simplifying} \quad 9C_{pri}\tilde{\rho}(A - D) \geq 2\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)}D + 2D^2 + 6C_{pri}\tilde{\rho}(A - D)$$

$$3C_{pri}\tilde{\rho}(A - D) \geq 2D\sqrt{D^2 + 3C_{pri}\tilde{\rho}(A - D)} + 2D^2 \quad (\text{A-9})$$

The last inequality is true since $D < 0$. Thus we have shown that the left hand side of the inequality in (A-8) is increasing in λ .

A.10 Proof of Lemma 6

An investor will be ready to invest to become a private investor iff $E(B_{pri}) \geq C$. Expanding B_{pri} we have:

$$Pr(Firm) * Pr(V_{pri}^I \geq V_{pub}^I) * (1 - \gamma)(\max(0, V_{pri}^I - V_{pub}^I)) \geq C \quad (\text{A-10})$$

In the above inequality, V_{pri}^I is strictly increasing in the ρ of the private investor. On the other hand V_{pub}^I is stochastic for the private investor and is independent of his ρ . Thus given an expected V_{pub}^I the payoff for the private investor is strictly increasing in his ρ . Since the right hand side is independent of ρ we have our result that private investors with ρ greater than the cut-off ρ_C will be willing to invest to become private investors. The cut-off is identified by the above inequality holding as an equality. Since $Pr(Firm)$ is increasing in the number of firms wanting to go private and V_{pub}^I is increasing in $\bar{\rho}$, we have our result that ρ_C is increasing in $\bar{\rho}$ and decreasing in N . Q.E.D.

A.11 Proof of Proposition 4

The expected cost of capital for the private firm can be given as the inverse of the expected valuation of the private investor. Thus the cost of capital is given as $CoC = E(1/V_{pri}^I)$. Thus CoC is decreasing in $E(V_{pri}^I)$. $E(V_{pri}^I)$ can be written as follows:

$$\begin{aligned}
E(V_{pri}^I) &= E\{(1 - \alpha)[e_{pri}^* \theta [\tilde{\rho} A + (1 - \tilde{\rho}) \eta_{pri}^* D - C_{pri} [\tilde{\rho} + (1 - \tilde{\rho}) \eta_{pri}^*] e_{pri}^*] - D]\} \\
&= E\{(1 - \alpha) \left[\frac{\alpha \theta}{\beta} [\tilde{\rho} + (1 - \tilde{\rho}) \eta] A \theta [\tilde{\rho} A + (1 - \tilde{\rho}) \eta D - \lambda L_{pri} [\tilde{\rho} + (1 - \tilde{\rho}) \eta] \frac{\alpha \theta}{\beta} [\tilde{\rho} + (1 - \tilde{\rho}) \eta] A] - D \right]\} \\
&= (1 - \alpha) \left\{ \frac{\alpha \theta^2 A}{\beta} E[\tilde{\rho} + (1 - \tilde{\rho}) \eta] [\tilde{\rho} A + (1 - \tilde{\rho}) \eta D] - \lambda L_{pri} \frac{\alpha^2 \theta^3 A^2}{\beta^2} E[\tilde{\rho} + (1 - \tilde{\rho}) \eta]^3 - D \right\}
\end{aligned}$$

We can show that the CoC is non-decreasing in the number of firms wanting to go private in two steps.

From Lemma 6 we know that the expected ρ of the private investor is decreasing in the number of firms wanting to go private. (Since the cut-off value of ρ , that is, ρ_C is decreasing in the number of firms wanting to go private, this implies that the expected value of ρ is also decreasing). Hence if we can show that the expected value of V_{pri}^I is decreasing in the expected value of ρ of the private investor we are done. To show this we will show that V_{pri}^I is increasing in $\tilde{\rho}$. This automatically implies that $E(V_{pri}^I)$ is increasing in $E(\tilde{\rho})$ and consequently the CoC is decreasing in $E(\tilde{\rho})$. One caveat that is important is that $E(V_{pri}^I)$ depends on the first moment of $\tilde{\rho}$, but also on the higher moments. These higher moments are also impacted by a shift in the cut-off ρ_C . Since the impact of higher moments will be of an order of magnitude lower than that of the first moment we choose to ignore them in the following proof. To show that V_{pri}^I is increasing in $\tilde{\rho}$ we simplify V_{pri}^I and then differentiate it with respect to $\tilde{\rho}$ and apply

the envelope theorem. To show $\frac{\partial V_{pri}^I}{\partial \tilde{\rho}} \geq 0$ it is sufficient to show that the following is greater than 0.

$$[A - \eta D] - \lambda L_{pri} \frac{2\alpha\theta A}{\beta} (1 - \eta)[\tilde{\rho} + (1 - \tilde{\rho})\eta] \geq 0$$

This condition is nothing but the rate of increase of the value of the option to invest with $\tilde{\rho}$. It is natural, that the value of the option should increase in ρ . A necessary and sufficient condition for that is that the liquidity penalty is not so large that it actually results in a reduction in the value of the investment option. This is a natural restriction on the size of λ . We assume this here. That is, we assume that λ is sufficiently small that the above condition holds. Thus V_{pri}^I is increasing in $\tilde{\rho}$. Consequently $E(V_{pri}^I)$ is increasing in $E(\tilde{\rho})$. Hence this along with our earlier argument shows that CoC is non-decreasing in the number of firms wanting to go private. Similarly since the expected probability of a successful going private transaction is increasing in $E(\tilde{\rho})$ we have the second part of the proposition. Q.E.D.

REFERENCES

1. Admati, Anat, Paul Pfleiderer and Josef Zechner (1994), “Large Shareholder Activism, Risk-Sharing, and Financial Market Equilibrium”, *Journal of Political Economy* 102, 1097-1130.
2. Allen, Franklin and Douglas Gale, (1999), “Diversity of Opinion and Financing of New Technologies” , *Journal of Financial Intermediation* 8, 68-89.
3. Bhattacharya, Sudipto and Giovanna Nicodano (2001), “Insider Trading, Investment and Liquidity: A Welfare Analysis”, *Journal of Finance*, 56, 1141-1156.
4. Bhide, Amar (1993), “The Hidden Costs of Stock Market Liquidity”, *Journal of Financial Economics* 34 (1), 31-51.
5. Bolton, Patrick and Ernst-Ludwign von Thadden (1998), “Blocks, Liquidity and Corporate Control”, *Journal of Finance* 53 (1), 1-26.
6. Burkart, Mike, Denis Gromb and Fausto Panunzi (1997), “Large Shareholders, Monitoring, and the Value of the Firm”, *Quarterly Journal of Economics*, 112, 693-728.
7. Chemmanur, Thomas, J. and Paolo Fulghieri (1999) “A Theory of the Going-Public Decision”, *Review of Financial Studies* 12 (2), 249-279.
8. Clarke, Ian and William Mackaness, (2001), Management Intuition: An Interpretative Account of Structure and Content of Decision Schemas Using Cognitive Maps, *Journal of Management*

- Studies* 38-2, 147-172.
9. Coffee, John, C. (1991), "Liquidity Versus Control: The Institutional Investor as Corporate Monitor", *Columbia Law Review* 91, 1277-1368.
 10. Daniel, Kent, David Hirshleifer and Avanidhar Subramaniam (1998), "Investor Psychology and Security Market under and Overreactions", *The Journal of Finance* 53 (6), 1839-1885.
 11. Diamond, Peter (1967), "The Role of the Stock Market in a General Equilibrium Model with Technological Uncertainty", *American Economic Review*, 57, 759-776.
 12. Jensen, Michael (1986), "Agency Costs of Free Cash flows, Corporate Finance and Takeovers", *American Economic Review* 76, 323-339.
 13. Gompers, Paul A., Joy L. Ishii, and Andrew Metrick, (2003) "Corporate Governance and Equity Prices", *Quarterly Journal of Economics*, 118-1, 107-155.
 14. Galton, Francis F.R.S. and Karl Pearson (1902), "The most suitable proportion between the value of first and second prices", *Biometrika*, 1-4, 385-399.
 15. Gibbons, Jean Dickinson (1971), "Nonparametric Statistical Inference", *McGraw-Hill Series in Probability and Statistics*.
 16. Grossman, Sandy and Oliver D. Hart (1980) "Take-Over Bids, the Free Rider Problem and the Theory of the Corporation", *Bell Journal of Economics and Management Science*, 42-64.
 17. Kaplan Steven N. and Per Stromberg (2003) "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts", *Review of Economic Studies* 70

- (2), 281-315.
18. Lehn, Kenneth and Annette Poulsen(1989), “Free cash flow and stockholder gains in going private transactions”, *The Journal of Finance* 44 (3), 771-787
 19. Lintner, John (1965), “Security Prices, Risk and Maximal Gains from Diversification”, *Journal of Finance*, 20, 587-615.
 20. Maksimovic, Vojislav and Pegaret Pichler (2001), “Technological Innovation and IPOs”, *Review of Financial Studies* 14, 459-494.
 21. Maug, Ernst (1998), “Large Shareholders as Monitors: Is there a Trade-off between Liquidity and Control?”, *Journal of Finance* 53 (1), 65-94.
 22. Pagano, Marco (1993), “The Flootation of Companies on the Stock Market: A Co-ordination Failure Model”, *European Economic Review* 37, 1101-1125.
 23. Ritter, Jay and Ivo Welch, (2002), “A review of IPO Activity, Pricing and Allocations”, *The Journal of Finance*, 57-4, 1795-1828.
 24. Ross, Stephen, A. (1976), “ The Arbitrage Theory of Capital Asset Pricing”, *Journal of Economic Theory*, 13, 341-360.
 25. Shah, Salman and Anjan V. Thakor (1988), “Private versus Public Ownership: Investment, Ownership Distribution, and Optimality”, *The Journal of Finance* 43 (1), 41-59. Structure,” *Journal of Political Economy*.

26. Zingales, Luigi (1995), "Insider Ownership and the Decision to Go Public", *Review of Economic Studies* 62, 425-448.

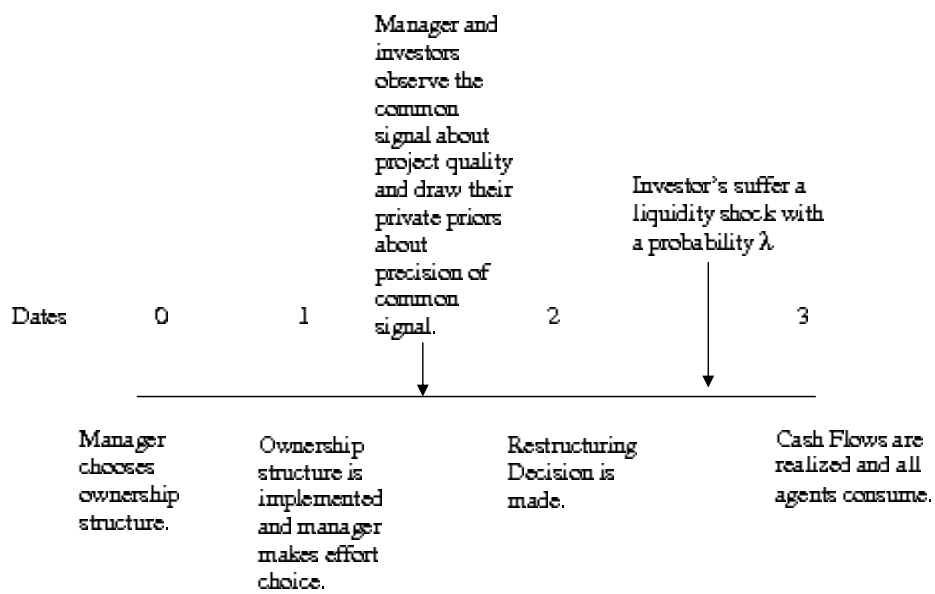


Figure 1: Sequence of Events